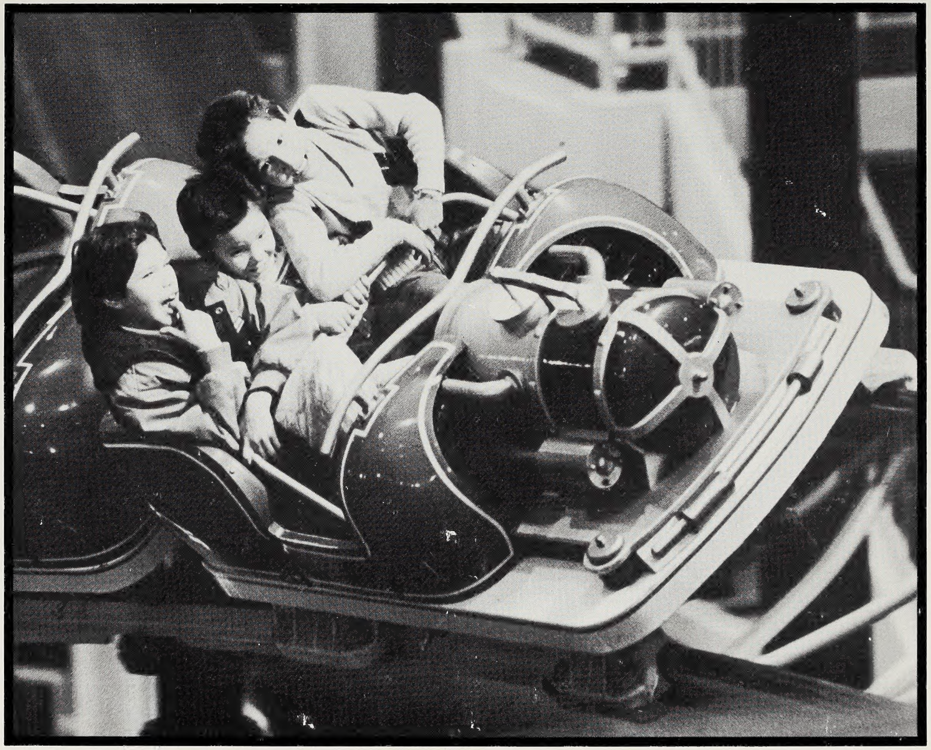
MATHEMATICS 9



Module 6: Measurement and Geometry

Distance Learning

Alberta EDUCATION

Mathematics 9

Module 6

MEASUREMENT AND GEOMETRY

Distance Learning

Alberta EDUCATION

This document is intended for	
Students	✓
Teachers (Mathematics 9)	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 9
Student Module
Module 6
Measurement and Geometry
Alberta Distance Learning Centre
ISBN No. 0-7741-0917-3

Cover Photo: WESTFILE INC.

Welcome to Module 6!

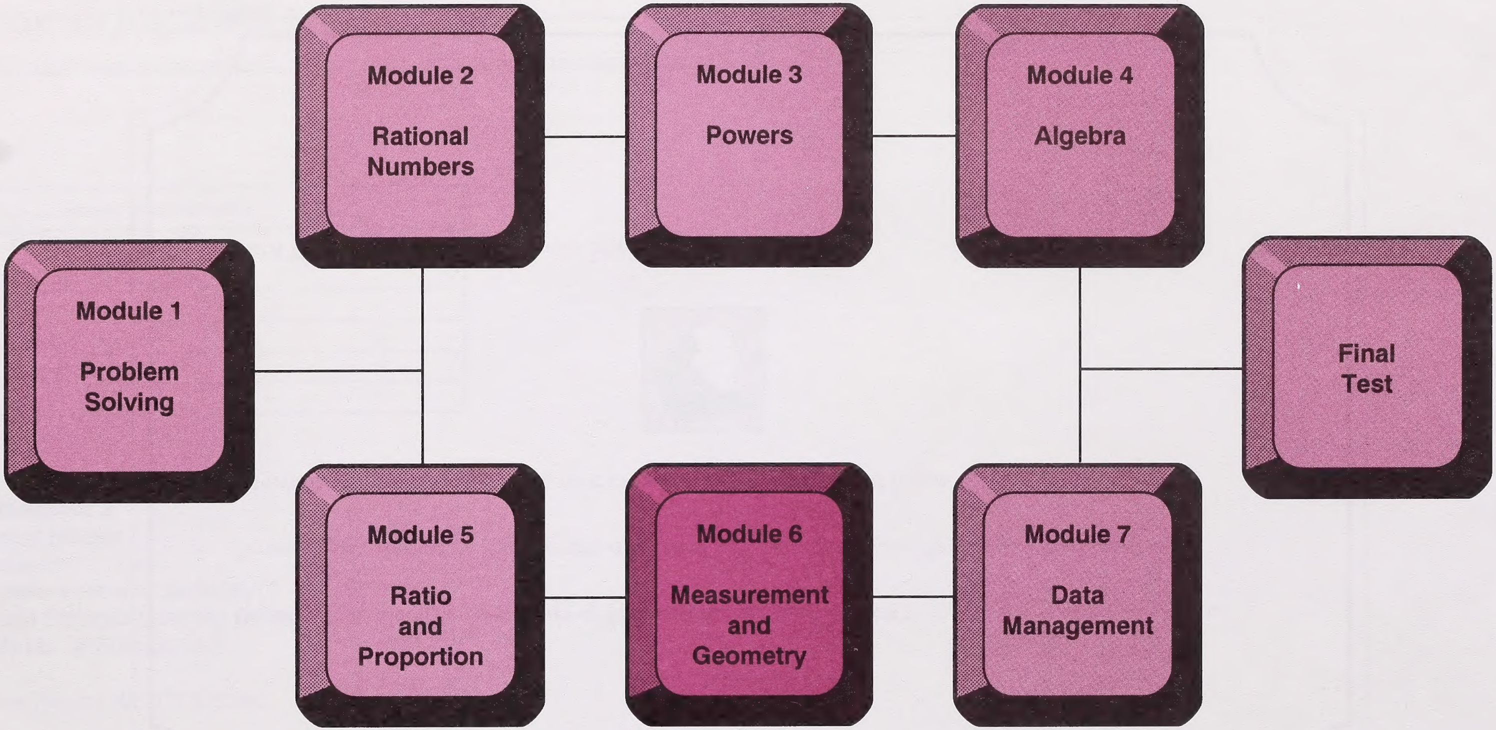
*We hope you'll enjoy your study of **Measurement and Geometry**.*

To make your learning a bit easier, a teacher will help guide you through the material.

So whenever you see this icon, turn on your audiocassette and listen.

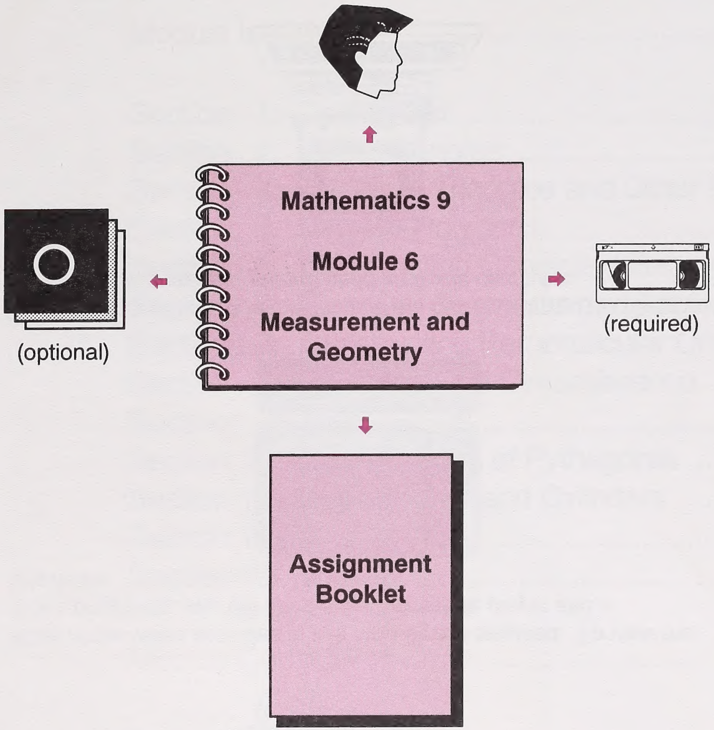
Turn the audiocassette on now to begin.

Course Overview

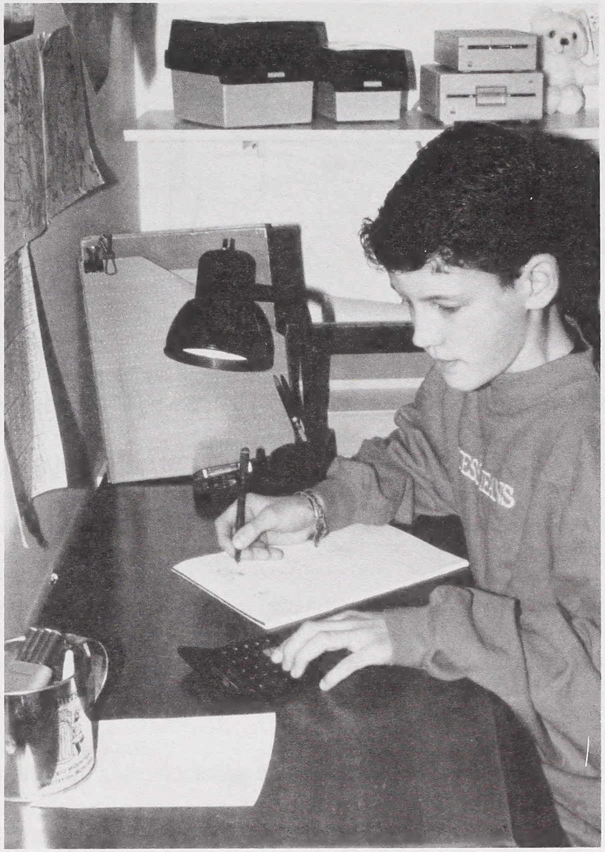


Mathematics 9 has seven modules and a final supervised test.

Module 6 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.



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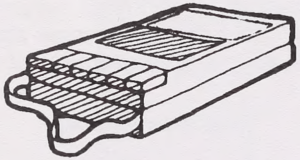
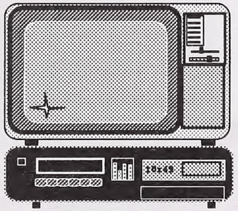
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Optional Equipment

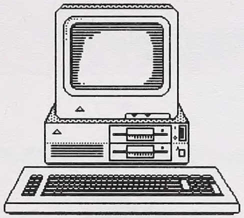
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



Most of the video activities in this module are required. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $\boxed{a\frac{b}{c}}$ key so that you can perform operations on fractions.

Evaluation

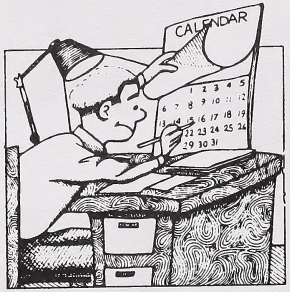
Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

Discuss how the module will be evaluated with your learning facilitator.

Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.



CONTENTS AT A GLANCE

MODULE INTRODUCTION

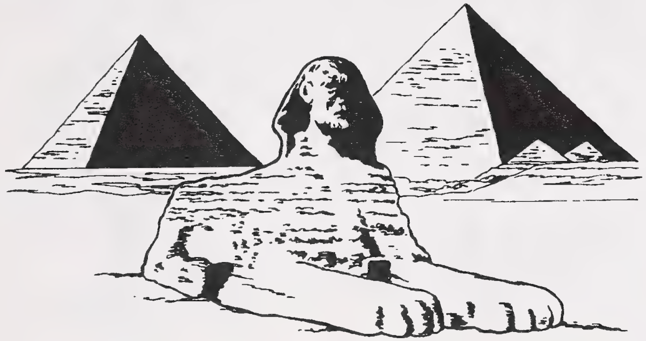
What Lies Ahead

In the module introduction you will preview Module 6.

Working Together

Geometry is one of the oldest branches of mathematics. The word *geometry* comes from the Greek words *geo* meaning "earth" and *metros* meaning "to measure".

Early Egyptian architects used measurement and geometry to design the pyramids.



Architects today also use measurement and geometry to design modern buildings.

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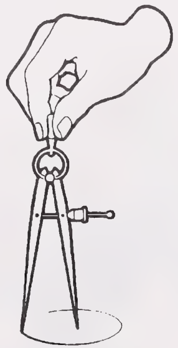
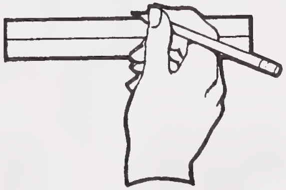
In this module you will review and extend your knowledge of the properties of lines, angles, and various polygons.

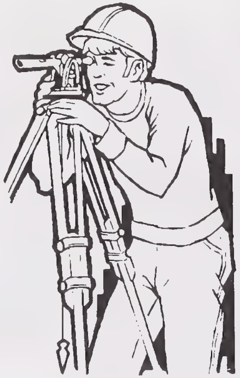
You will learn how to accomplish several geometric constructions using only two tools: a compass and a straightedge.



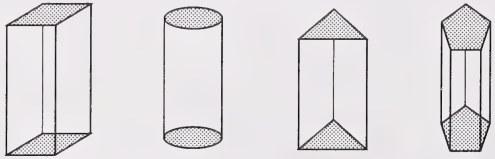


These types of constructions go back many centuries, but they are still very important to people working in the fields of drafting and architecture today.

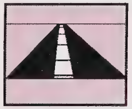
You will review how to calculate the perimeter and area of various polygons and the circumference and area of circles. You will also learn about the theorem of Pythagoras which has important applications today. For example, surveyors use the theorem of Pythagoras to calculate certain distances.



Finally you will investigate right prisms and cylinders and learn how to calculate the surface area and volume of these three-dimensional shapes.



GETTING SET



What Lies Ahead

In this section you will review skills previously developed in Mathematics 7 and 8.

- classifying geometric figures as rays, lines, segments, angles, and polygons
- classifying an angle according to its measure
- classifying pairs of lines as parallel, perpendicular, or intersecting
- classifying a polygon according to the number of sides and angles
- classifying a triangle according to the measure of its sides and angles, the number of lines of symmetry, and the turn order
- classifying a quadrilateral by the number of parallel sides, the presence of four congruent sides, the presence of four right angles, the presence of four right angles and four congruent sides, the number of lines of symmetry, and the turn order

You will also pretest the skills taught in this module.

Review

1. Match each of the geometric figures in Column A with the correct name from Column B.

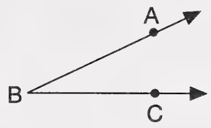
Column A

a. 

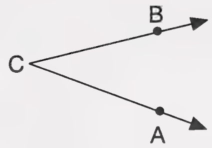
b. 

c. • A

d. 

e. 

f. 

g. 

Column B

i. A (point A)

ii. $\angle ABC$ (angle ABC)

iii. $\overline{AB}$ (segment AB)

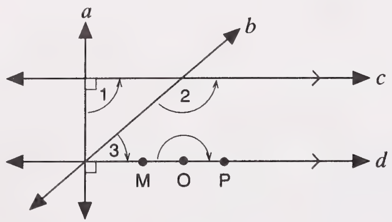
iv. $\overrightarrow{AB}$ (ray AB)

v. $\overleftrightarrow{AB}$ (line AB)

vi. $\angle BCA$ (angle BCA)

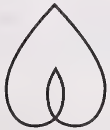
vii. $\overrightarrow{BA}$ (ray BA)

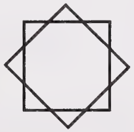
2. State whether each of the following statements is **true** or **false**.

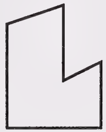


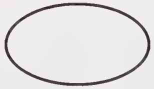
a. Line a and line b are parallel lines.
b. Line a and line b are perpendicular lines.
c. Line a and line b are intersecting lines.
d. Line a and line d are parallel lines.
e. Line a and line d are perpendicular lines.
f. Line a and line d are intersecting lines.
g. Line c and line d are parallel lines.
h. Line c and line d are perpendicular lines.
i. Line c and line d are intersecting lines.
j. $\angle 1$ is an acute angle.
k. $\angle 1$ is a straight angle.
l. $\angle 2$ is a right angle.
m. $\angle 2$ is an obtuse angle.
n. $\angle 3$ is an acute angle.
o. $\angle 3$ is a reflex angle.
p. $\angle$MOP is an obtuse angle.
q. $\angle$MOP is a straight angle.

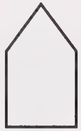
3. Which of the following figures are polygons? Explain why.

A. 

B. 

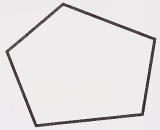
C. 

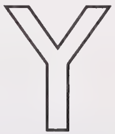
D. 

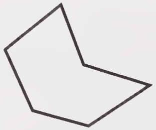
E. 

F. 

4. Choose the name that represents the number of sides and number of angles shown for each of the following polygons.

a. 

b. 

c. 

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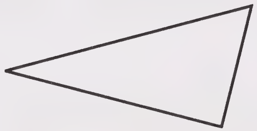
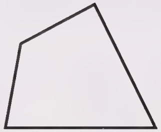
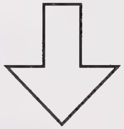
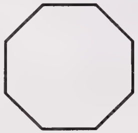
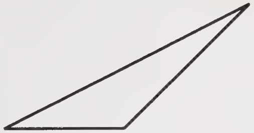
e. 

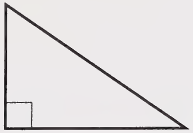
f. 

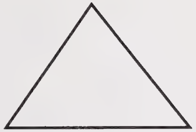
g. 

h. 

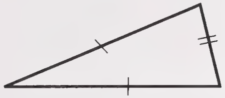
5. Classify each of the following triangles according to the measure of its largest angle.

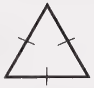
a.


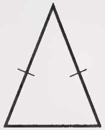
b.


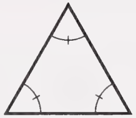
c.


6. Classify each of the following triangles according to the measures of its sides or angles.

a.


b.


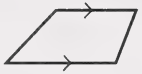
c.


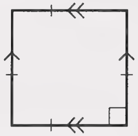
d.


7. Copy and complete the following table in your notebook. Write **N/A** for not applicable if the triangle does not have flip or turn symmetry.

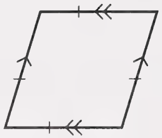
Kind of Triangle	Number of Lines of Symmetry	Order of Turn Symmetry
Equilateral		
Isosceles		
Scalene		

8. Choose the best name for each of the following quadrilaterals.

a.


b.


c.


d.


e.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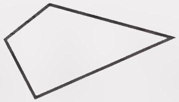

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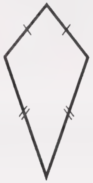

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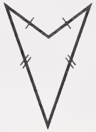
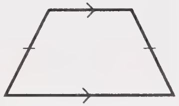


i.


9. Copy and complete the following table in your notebook. Write **N/A** for not applicable if the quadrilateral does not have flip symmetry or turn symmetry.

Kind of Quadrilateral	Number of Lines of Symmetry	Order of Turn Symmetry
Quadrilateral		
Kite		
Deltoid		
Trapezoid		
Isosceles Trapezoid		
Parallelogram		
Rhombus		
Rectangle		
Square		

10. Copy and complete the following table in your notebook. Use the words **yes** or **no** to answer.

Kind of Quadrilateral	Are Diagonals Congruent?	Do Diagonals Bisect Each Other?	Do Diagonals Bisect Each Other at 90° Angles?
Quadrilateral			
Kite			
Trapezoid			
Isosceles Trapezoid			
Parallelogram			
Rhombus			
Rectangle			
Square			

Turn to the Appendix to check your answers.

Working Together

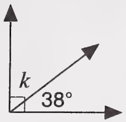
The Pretest in this section will help you and your learning facilitator to determine your strengths and weaknesses and individualize your learning plan.

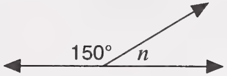
Pretest

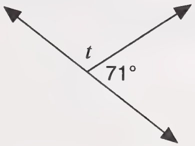
1. Define each of the following terms.

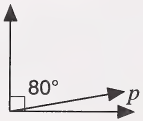
   a. complementary angles
   b. supplementary angles
   c. adjacent angles of intersecting lines
   d. opposite angles of intersecting lines

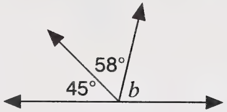
2. Calculate the measure of each of the missing angles. **Do not** use a protractor.

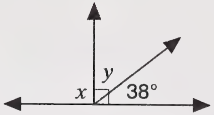
   a.


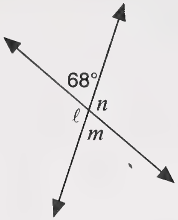
   b.


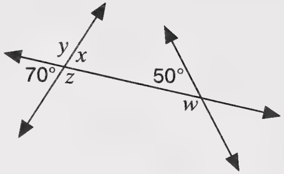
   c.


   d.


   e.


   f.


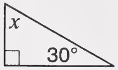
   g.


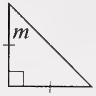
   h.


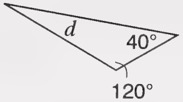
3. What is the sum of the angles of each of these polygons?

   a. triangle
   b. quadrilateral
   c. pentagon
   d. hexagon
   e. heptagon
   f. octagon

4. Calculate the measure of the missing angles. **Do not** use a protractor.

   a.


   b.


   c.


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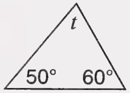
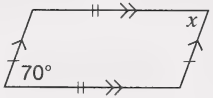
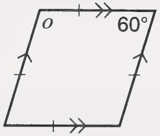
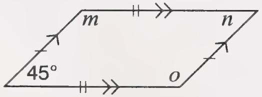


   e.


   f.


   g.


5. How many congruent triangles are in each of the following regular polygons?

a. quadrilateral   b. pentagon   c. hexagon

6. How many lines of symmetry are in each of the following regular polygons?

a. regular pentagon   b. regular hexagon
c. regular heptagon

7. What is the order of turn symmetry in each of the following regular polygons?

a. regular pentagon   b. regular hexagon
c. regular heptagon

8. Use a compass, protractor, and straightedge to draw a regular dodecagon (12-sided polygon).

9. a. Use a ruler to draw a 6-cm segment on your paper. Then, using only a compass and a straightedge, construct a congruent segment.

b. Use a protractor to draw a 70° angle on your paper. Then, using only a compass and a straightedge, construct a congruent angle.

10. Draw a 8-cm segment on your paper. Then, using only a compass and a straightedge, construct an equilateral triangle with sides of 8 cm.

11. a. Use a ruler to draw a 9-cm segment. Then, using only a compass and a straightedge, bisect the segment.

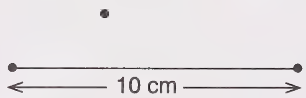
b. Use a protractor to draw an angle of 82°. Then, using only a compass and a straightedge, bisect the angle.

12. Use only a compass and a straightedge to construct angles with the following measures.

a. 90°   b. 45°   c. 60°   d. 30°

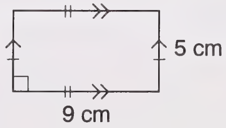
13. a. Use a ruler to draw a segment of 8.5 cm. Put a dot to represent a point 2 cm from the endpoint of the line segment. Then, using only a compass and a straightedge, construct a line through the point that is perpendicular to the segment.

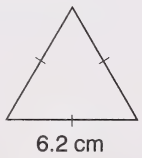
b. Use a ruler to draw a 10-cm segment. Put a dot to represent a point above the line segment as shown.

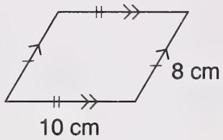


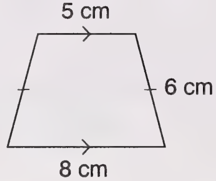
Use only a compass and a straightedge to draw a line through the point that is perpendicular to the segment.

14. Calculate the perimeter of each of these polygons.

a.


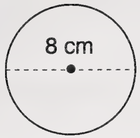
b.


c.


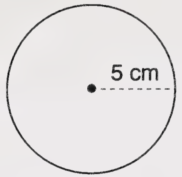
d.


15. Calculate the circumference of each of these circles.

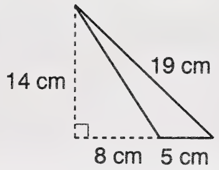
a.



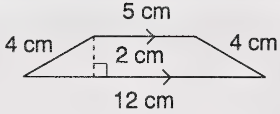
b.



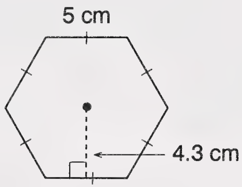
c.



d.

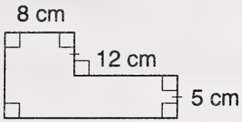


e.

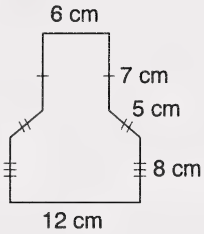


16. Calculate the perimeter of each of these figures.

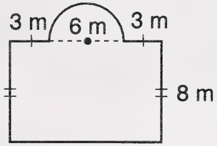
a.



b.



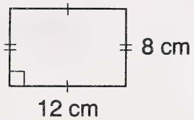
c.



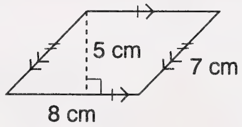
17. The length of a rectangle is two more than three times its width. If the perimeter of the rectangle is 92 m, what are the dimensions of the rectangle?

18. Calculate the area of each of these polygons.

a.

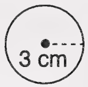


b.

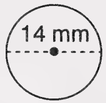


19. Calculate the area of each of these circles.

a.

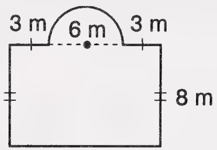


b.

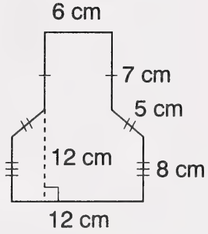


20. Calculate the area of each of these figures.

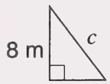
a.



b.



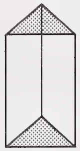
21. Find the lengths of the missing sides.

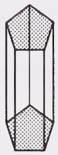
a. 

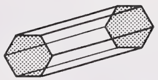
b. 

22. Name the following three-dimensional figures.

a. 

b. 

c. 

d. 

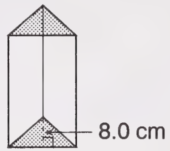
e. 

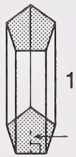
23. Sketch the net of an octagonal prism.

24. Calculate the surface area of each of these three-dimensional figures. **Hint:** The bases in c. and d. are regular polygons.

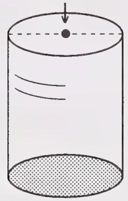
a. 

b. 

c. 

d. 

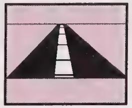
25. Calculate the surface area of the exterior of this drinking glass.



26. Calculate the volumes of the three-dimensional figures in Question 24.

See your learning facilitator to check your answers and to receive further instructions.

PAIRS OF ANGLES



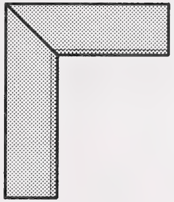
What Lies Ahead

In this section you will learn the following ways to classify pairs of angles.

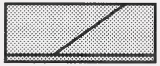
- supplementary angles
- complementary angles
- adjacent angles
- opposite angles

Working Together

Knowledge about angles is important in many jobs in the everyday world. For example, carpenters need to know about angles when they mitre corners of door frames or join baseboards.

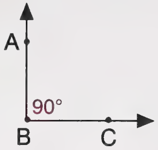


Mitred corner of door frame



Joined baseboard

Two important angles are the right angle and the straight angle. A right angle has a measure of 90° and a straight angle has a measure of 180°.

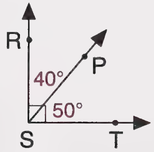


Right angle

Straight angle

Two angles that have a sum of 90° are **complementary angles**.

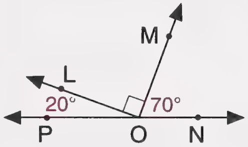
Example 1



$40° + 50° = 90°$

$\angle RSP$ and $\angle PST$ are complementary angles.

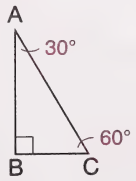
Example 2



$20° + 70° = 90°$

$\angle LOP$ and $\angle MON$ are complementary angles.

Example 3

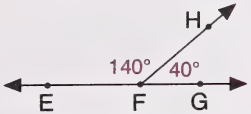


$30° + 60° = 90°$

$\angle A$ and $\angle C$ are complementary angles.

Two angles that have a sum of 180° are **supplementary angles**.

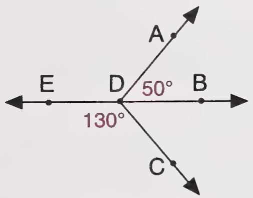
Example 1



$140° + 40° = 180°$

$\angle EFH$ and $\angle HFG$ are supplementary angles.

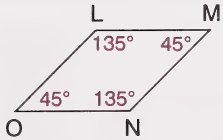
Example 2



$50° + 130° = 180°$

$\angle ADB$ and $\angle EDC$ are supplementary angles.

Example 3

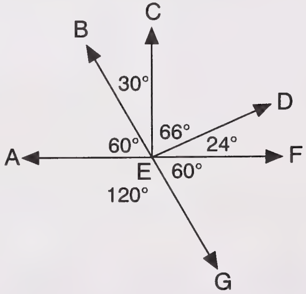


$135° + 45° = 180°$

$\angle L$ and $\angle O$ are supplementary angles.
$\angle L$ and $\angle M$ are supplementary angles.
$\angle N$ and $\angle O$ are supplementary angles.
$\angle N$ and $\angle M$ are supplementary angles.

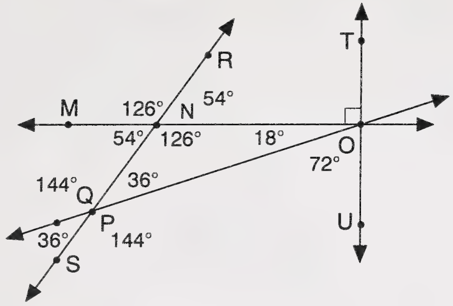
Practice Activity 1

1. Complete each of the following statements.



a. $\angle AEB$ and ________ are complementary angles.
b. $\angle AEB$ and ________ are supplementary angles.
c. $\angle CED$ and ________ are complementary angles.
d. $\angle FEG$ and ________ are supplementary angles.

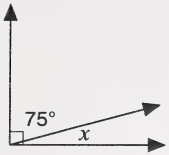
2. Complete each of the following statements.



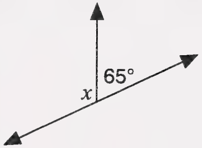
a. ∠RNO and ________ are supplementary angles.
b. ∠RNO and ________ are complementary angles.
c. ∠QPS and ________ are supplementary angles.
d. ∠MOP and ________ are complementary angles.

3. Calculate the measures of the unknown angles. **Do not** measure the angles with a protractor.

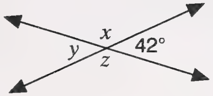
a.



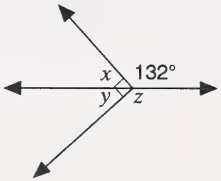
b.



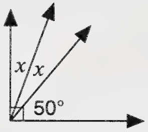
c.



d.



e.



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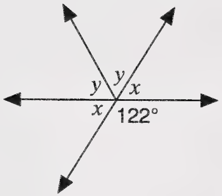
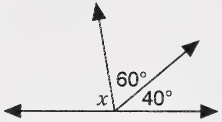
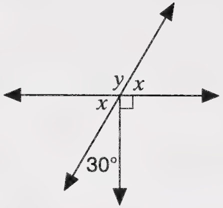


g.



h.



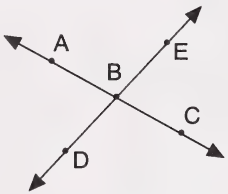
Turn to the Appendix to check your answers.

Working Together

Certain pairs of angles have names that describe their position in relation to each other.

When two lines intersect, four angles are formed. The angles that are next to each other and share a common vertex and ray are **adjacent angles**. The nonadjacent angles are **opposite angles**.

Example

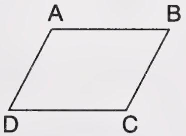


$\angle ABE$ and $\angle EBC$ are adjacent angles.
$\angle EBC$ and $\angle CBD$ are adjacent angles.
$\angle CBD$ and $\angle DBA$ are adjacent anlges.
$\angle DBA$ and $\angle ABE$ are adjacent angles.
$\angle ABE$ and $\angle DBC$ are opposite angles.
$\angle EBC$ and $\angle ABD$ are opposite angles.

Note: Opposite angles may be called **vertically opposite angles** or **vertical angles**.

The two angles of a quadrilateral whose vertices are adjacent are **adjacent angles**. The two nonadjacent angles of a quadrilateral are **opposite angles**.

Example

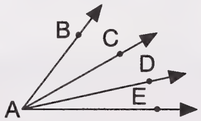


$\angle A$ and $\angle B$ are adjacent angles.
$\angle B$ and $\angle C$ are adjacent angles.
$\angle C$ and $\angle D$ are adjacent angles.
$\angle D$ and $\angle A$ are adjacent angles.
$\angle A$ and $\angle C$ are opposite angles.
$\angle B$ and $\angle D$ are opposite angles.

Practice Activity 2

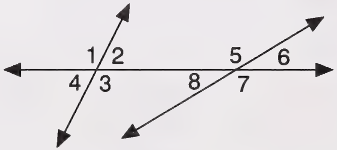
Print Alternative

1. Name the angle that is adjacent to each of the following angles.



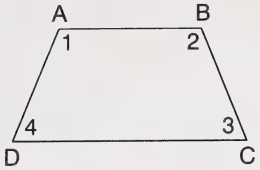
a. $\angle BAC$ **b.** $\angle BAD$ **c.** $\angle DAC$ **d.** $\angle EAC$

2. Complete each of the following statements.



a. $\angle 1$ and _____ are adjacent angles.
b. $\angle 1$ and _____ are opposite angles.
c. $\angle 8$ and _____ are adjacent angles.
d. $\angle 8$ and _____ are opposite angles.

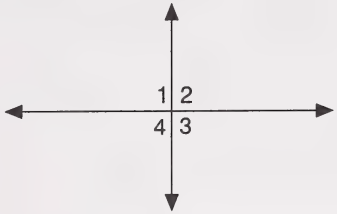
3. Complete each of the following statements.



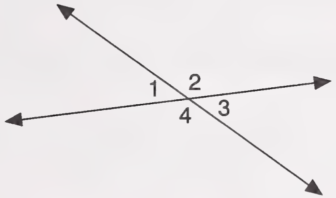
a. $\angle 2$ and _____ are adjacent angles.
b. $\angle 2$ and _____ are opposite angles.
c. $\angle 3$ and _____ are adjacent angles.
d. $\angle 3$ and _____ are opposite angles.

4. Use a protractor to measure the angles formed by the following pairs of intersecting lines.

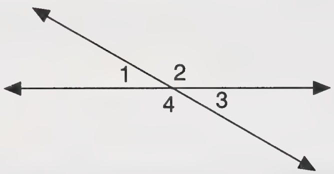
a.



b.



c.

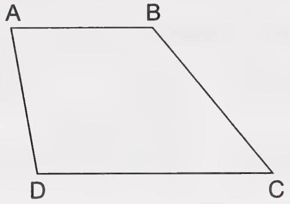


5. a. What do you notice about each pair of opposite angles in Question 4?

b. What do you notice about each pair of adjacent angles in Question 4?

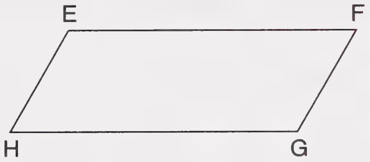
6. Use a protractor to measure the angles in the following convex quadrilaterals.

a.



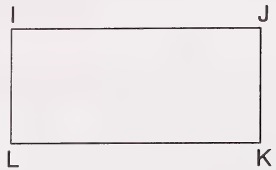
Polygon ABCD is a trapezoid.

b.



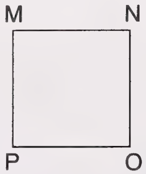
Polygon EFGH is a parallelogram.

c.



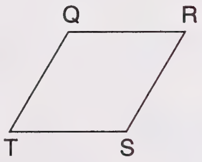
Polygon IJKL is a rectangle.

d.



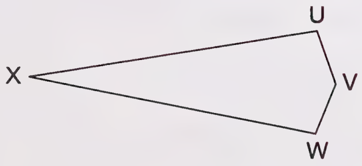
Polygon MNOP is a square.

e.



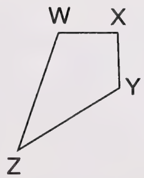
Polygon QRST is a rhombus.

f.



Polygon UVWX is a kite.

g.



Polygon WXYZ is a quadrilateral.

7. a. In which quadrilaterals in Question 6 are all pairs of opposite angles congruent?

   b. In which quadrilaterals in Question 6 are some pairs of opposite angles congruent?

   c. In which quadrilaterals in Question 6 are no pairs of opposite angles congruent?

   d. In which quadrilaterals in Question 6 are all pairs of adjacent angles supplementary?

   e. In which quadrilaterals in Question 6 are some pairs of adjacent angles supplementary?

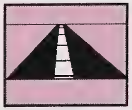
   f. In which quadrilaterals in Question 6 are no pairs of adjacent angles supplementary?

Computer Alternative

9. Work through Program 11, "Learning All the Angles", on Disk C of *Mathematics Courseware 6* (Houghton Mifflin).

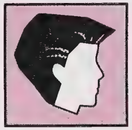
Turn to the Appendix to check your answers.

ANGLES IN TRIANGLES AND OTHER POLYGONS



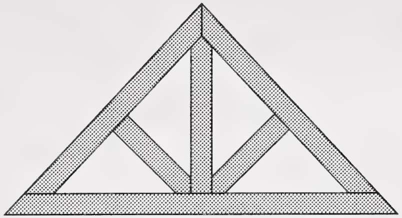
What Lies Ahead

In this section you will determine the sum of the angles of a triangle and the sum of the angles in any polygon.



Working Together

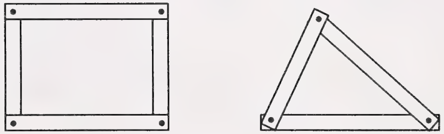
This structure is used to support the roof of a building.



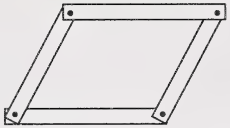
The large triangle is divided into smaller triangles. How many triangles are there altogether?

The triangular design is used because triangles are rigid.

To discover what this means, make a rectangle and a square with strips of paper and paper fasteners.



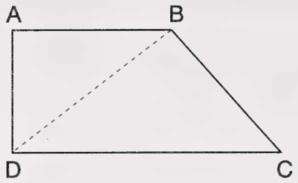
You will discover that the angles of the rectangle can be changed.



The angles of the triangle, on the other hand, cannot be changed.

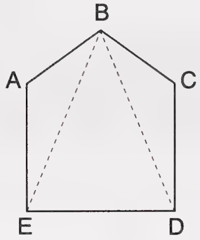
Other polygons can be made rigid by adding diagonals.

Example 1



The diagonal $\overline{BD}$ makes the quadrilateral ABCD rigid. Notice that two triangles are formed.

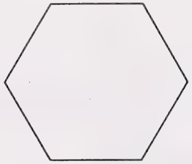
Example 2

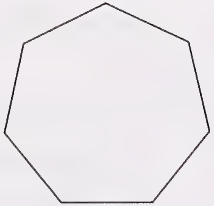


The diagonals $\overline{BE}$ and $\overline{BD}$ make the pentagon ABCDE rigid. Notice that three triangles are formed.

Practice Activity 1

1. Trace these polygons into your notebook and draw the diagonals that make each of the following polygons rigid.

a. 

b. 

2. a. Copy and complete the following table in your notebook.

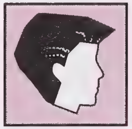
Polygon	Number of Sides	Number of Diagonals Needed to Make the Polygon Rigid
Triangle		
Quadrilateral		
Pentagon		
Hexagon		

b. Look for a pattern in the complete table in Question 2.a. Write a formula to show the relationship between the number of sides (n) and the number of diagonals (d).

c. Use the formula in 2.b. to complete the following table in your notebook.

Polygon	Number of Sides	Number of Diagonals Needed to Make the Polygon Rigid
Heptagon		
Octagon		
Nonagon		
Decagon		

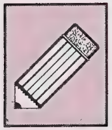
Turn to the Appendix to check your answers.



Working Together

What is the sum of the measures of the angles in a triangle? You will discover the answer to this question in the next activity.

Practice Activity 2



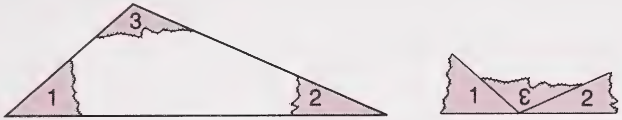
Print Alternative

1. Draw several triangles that are of different sizes and shapes.

   Cut out each triangle and carefully tear off the three corners of each. Then place the pieces side by side.

   Notice that the three angles of each triangle form a straight angle.

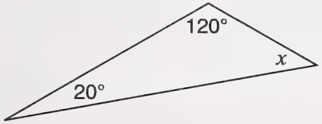
   Example



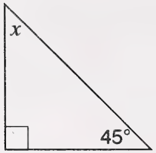
   What is the sum of the measures of the angles of a triangle?

2. Use your answer for Question 1 to find the measure of the missing angle in each of the following triangles.

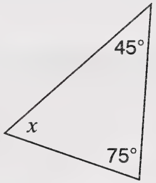
a.



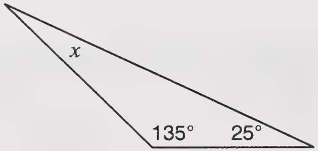
b.

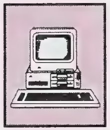


c.



d.

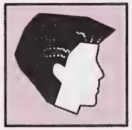




Computer Alternative

3. For extra practice, you may wish to complete Program 11, "Triangle Tryouts" on Disk C of *Mathematics Activities Courseware 8* (Houghton Mifflin).

Turn to the Appendix to check your answers.

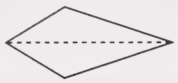


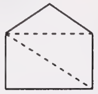
Working Together

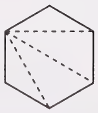
What is the sum of the measures of a quadrilateral? a pentagon? a hexagon? a heptagon? an octagon? another polygon? You will discover the answers to these questions in the next activity.

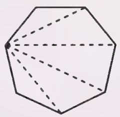
Practice Activity 3

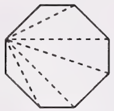
1. How many triangles are formed by the diagonals in each of the following polygons?

a. 

b. 

c. 

d. 

e. 

2. **a.** Copy and complete the following table in your notebook.

Kind of Polygon	Number of Sides	Number of Triangles	Sum of the Measures of the Angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		

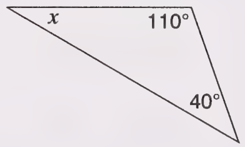
b. Look for a pattern in the completed table in 2.a. Write a formula to show the relationship between the number of sides (n) and the number of triangles (t).

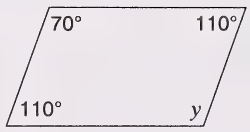
c. Write a formula to show the relationship between the number of sides (n) and the sum of the measures of the angles (a) in a polygon.

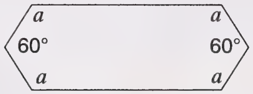
3. Use the formula from Question 2.c. to calculate the sum of the angles in each of the following polygons.

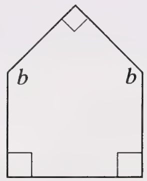
a. nonagon **b.** decagon **c.** dodecagon

4. Calculate the measure of the missing angles in each of the following polygons. **Do not** use a protractor.

a. 

b. 

c. 

d. 

Turn to the Appendix to check your answers.

REGULAR POLYGONS

What Lies Ahead

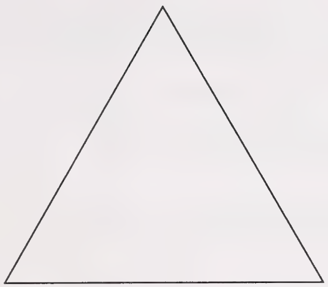
In this section you will discover several properties of regular poygons.

You will also make regular polygons in several ways.

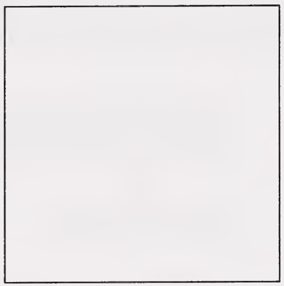
- folding paper
- using a compass, straightedge, and protractor
- using a computer and the computer language LOGO

Working Together

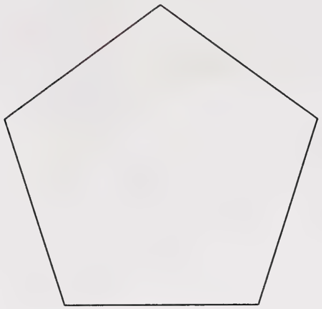
Regular polygons have congruent sides and congruent angles. Here are several regular polygons.



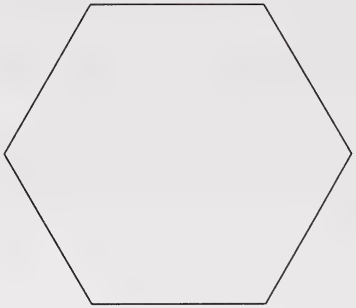
Regular triangle



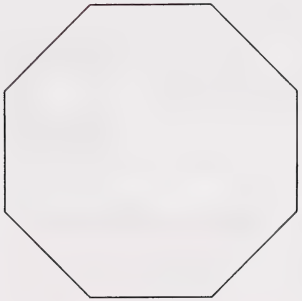
Regular quadrilateral



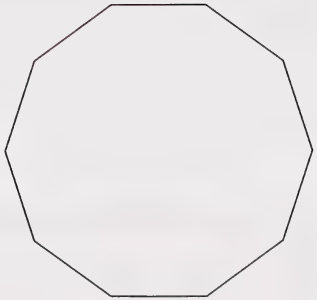
Regular pentagon



Regular hexagon



Regular octagon



Regular decagon

Practice Activity 1

1. Use a protractor to measure the angles of each of the regular polygons on this page.

2. Trace each of the regular polygons from the previous page onto your own paper and cut out the tracings.

   a. Does each of the regular polygons have flip symmetry? **Hint:** If a polygon has flip symmetry, you will be able to fold one-half of the polygon over the other half.

   b. How many lines of symmetry does each regular polygon have? **Hint:** Each fold line is a line of symmetry.

   c. Write a formula to describe the relationship between the number of sides (n) of a regular polygon and the number of lines of symmetry (ℓ).

3. Colour one angle in each of the tracings of the regular polygons. Place each of the tracings over the original on the previous page and turn it about the turn centre (the point at which the lines of symmetry cross). Test to see if each of the regular polygons has turn symmetry. **Hint:** If a polygon has turn symmetry, it will fit onto itself more than once in a full turn.

   a. How many times does each polygon fit onto itself in one full turn? This is called the turn order.

   b. Write a formula to describe the relationship between the number of sides of the regular polygon (n) and the turn order (o).

4. Use the results of Questions 2 and 3 to make the following predictions.

   a. Does a regular heptagon have flip symmetry? If so, how many lines of symmetry does it have?

   b. Does a regular heptagon have turn symmetry? If so, what is its turn order?

   c. Does a regular nonagon have flip symmetry? If so, how many lines of symmetry does it have?

   d. Does a regular nonagon have turn symmetry? If so, what is its turn order?

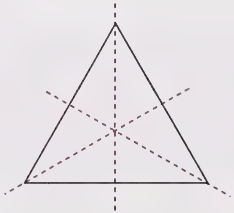
5. Glue or tape the tracings onto a piece of paper. Test to see if you can **circumscribe** each regular polygon. That is, can you put the point of your compass on the point of symmetry and draw a circle about the polygon with every vertex of the polygon touching the circle?

Turn to the Appendix to check your answers.

Working Together

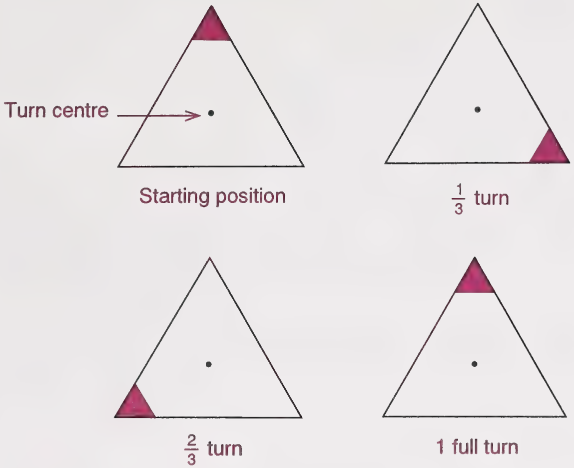
In Practice Activity 1 you learned that a regular polygon has flip symmetry and turn symmetry.

Example



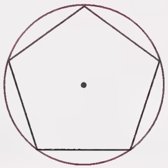
A regular triangle has three lines of symmetry.

A regular triangle has turn symmetry of order 3. It will fit onto itself three times in one full turn.



You also learned that a circle can circumscribe a regular polygon.

Example

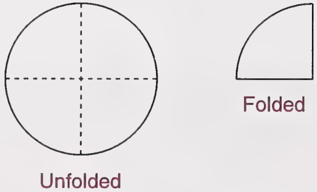


You can use these properties to help you make regular polygons.

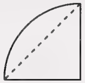
Practice Activity 2

For this activity you will need three circles cut from paper. Be sure the circles are not smaller than 10 cm in diameter.

1. Fold a circle into quarters.

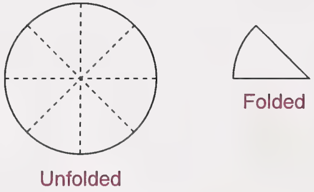


Then make a fold along the fold line shown in this diagram.

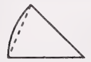


Unfold the circle. Name the regular polygon that is produced by the fold lines.

2. Fold a second circle in eighths.

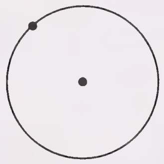


Then make a fold along the fold line shown in this diagram.

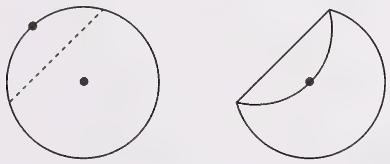


Unfold the circle. Name the regular polygon produced by the fold lines.

3. Mark the centre of the circle and a point on the edge of the circle.



Fold the paper so that the two points touch.

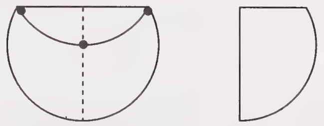


Unfolded

Folded

Mark two more points as shown in the diagram.

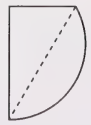
Fold so that the points touch.



Unfolded

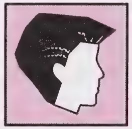
Folded

Fold along the fold line shown in the diagram.



Unfold the circle. Name the regular polygon produced by the fold lines.

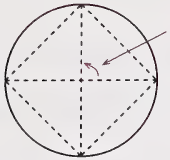
Turn to the Appendix to check your answers.



Working Together

In Practice Activity 2 you made several regular polygons.

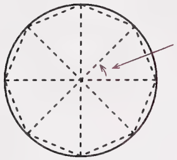
Look at the regular quadrilateral, or square, that you made by folding. Notice that there are four congruent angles formed at the centre of the circle. These are called **central angles**.



$360 \div 4 = 90$

So, each central angle has a measure of 90°.

Look at the regular octagon that you made by folding. Notice that there are eight congruent central angles.



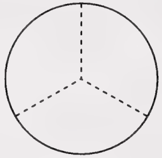
$360 \div 8 = 45$

So, each central angle has a measure of 45°.

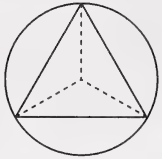
You can use this information to make regular polygons using a compass, a protractor, and a straightedge.

Practice Activity 3

1. Draw a circle with a compass. Make three congruent angles at the centre with a protractor as show in the diagram.



Use a straightedge to make a regular triangle as shown.



Check the accuracy of your drawing with a ruler and protractor.

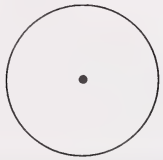
2. What will be the measure of each of the central angles for each of the following regular polygons?

a. regular pentagon
b. regular hexagon
c. regular nonagon
d. regular decagon

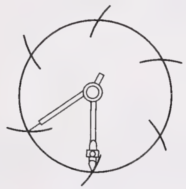
3. Draw each of the regular polygons listed in Question 2. Use a method similar to that described in Question 1. Check the accuracy of your drawings with a ruler and protractor.

4. Measure the length of the radius and the length of each side of the regular hexagon that you drew in Question 3. What do you notice?

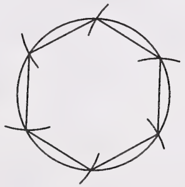
5. Draw a circle with a compass.



With the same compass setting, draw six arcs as shown in the diagram.



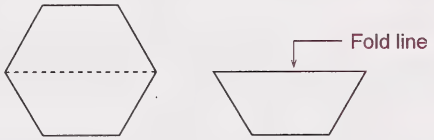
Connect the points where the arcs cut the circle with a straightedge.



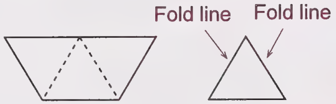
What kind of regular polygon is produced?

6. Use a similar method to that described in Question 5 to make a regular triangle. **Hint:** Connect every other point where the arcs cut the circle. Check the accuracy of your drawing with a ruler and a protractor.

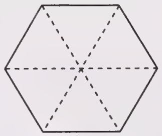
7. Cut out the figure constructed in Question 5. Fold the figure in half as shown in the diagram.



Make two more folds as show in the diagrams.



Then unfold the paper.



Are the six triangles formed by the fold lines congruent?

8. How many congruent triangles can be formed in each of these regular polygons?

a. pentagon
b. heptagon
c. octagon
d. decagon

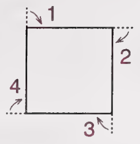
Turn to the Appendix to check your answers.

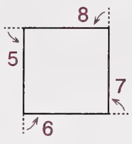
Working Together

This part of the section is included for enrichment. If you decide to do this part of the section, you will need a computer and the computer programming language, LOGO.

If you travel around in a clockwise or counter clockwise direction, you will make a complete trip of 360°.

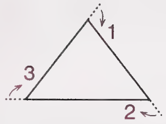
Example 1

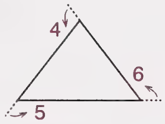




$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ $\quad$ $\angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

Example 2





$\angle 1 + \angle 2 + \angle 3 = 360°$ $\quad$ $\angle 4 + \angle 5 + \angle 6 = 360°$

You can use this knowledge to draw regular polygons on a computer with the LOGO language.

Practice Activity 4

1. Use a computer and the LOGO language to draw each of the following regular polygons. Use only the commands FD and RT.

   a. triangle **b.** square **c.** pentagon
   d. hexagon **e.** octagon

2. Use a computer and the LOGO language to draw each of the regular octagons in Question 1. This time use the REPEAT command.

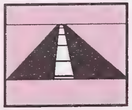
Turn to the Appendix to check your answers.

Did You Know?

Dr. Seymour Papert

During the late 1960s and early 1970s, Dr. Seymour Papert, working with a group of scientists, created turtle graphics and the computer programming language, LOGO. Dr. Papert got the idea for LOGO after watching a computer direct a pen, mounted in an apparatus that looked like an upside down bowl or turtle, to draw a picture.

CONSTRUCTING CONGRUENT SEGMENTS AND ANGLES



What Lies Ahead

In this section you will learn these skills.

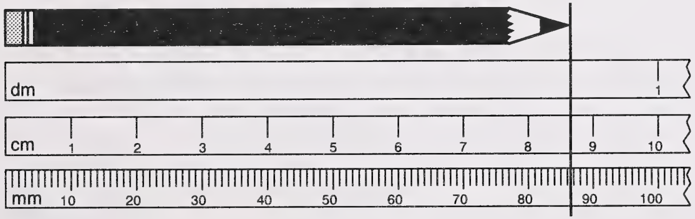
- copying segments
- copying angles
- constructing triangles by copying segments
- copying triangles

Working Together

Accuracy in measurement is very important in many situations in the everyday world.

Most measuring tools have scales with subdivisions. When you measure, you must compare the object to the scale and decide which subdivision on the scale the object is closest to. The accuracy of the measurement depends on the tool you use and how well you use it.

Some rulers, for example, may be divided into decimetres. Others are divided into centimetres or millimetres.



The pencil's length can be read on each scale.

- 0.9 dm
- 8.7 cm
- 86.5 mm

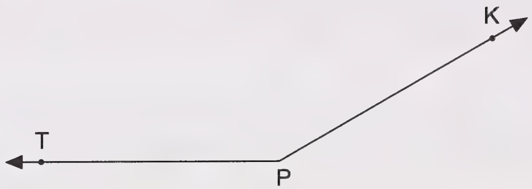
All the measurements are estimates, but the smaller the divisions on the scale, the more accurate the measurement.

Note: It is helpful to use clear plastic rulers rather than wooden rulers. You will be able to read the scale easier, and your measurements will be more accurate.

A protractor is a geometric tool used to measure angles. It is subdivided into degrees. When using a protractor, the accuracy of your measurement will depend on how you use the protractor.

Example

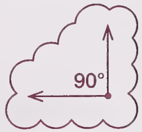
What is the measurement of $\angle KPT$?



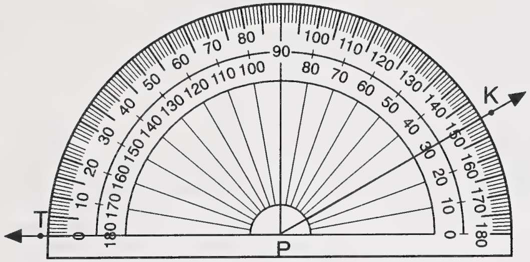
Solution

First decide whether the angle is greater than or less than 90°.

The angle is greater than 90°.



Then place the protractor on the angle. Be sure the base line of the protractor is on one arm of the angle. Be sure the centre of the protractor is at the vertex of the angle.



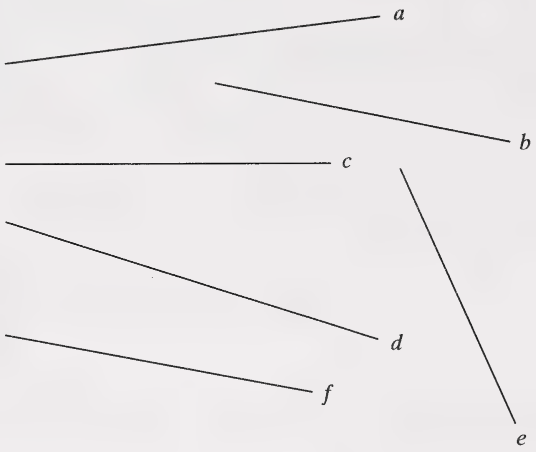
Decide whether to read the inner or outer scale. Is the correct measurement 150° or 30°?

$\angle KPT = 150°$

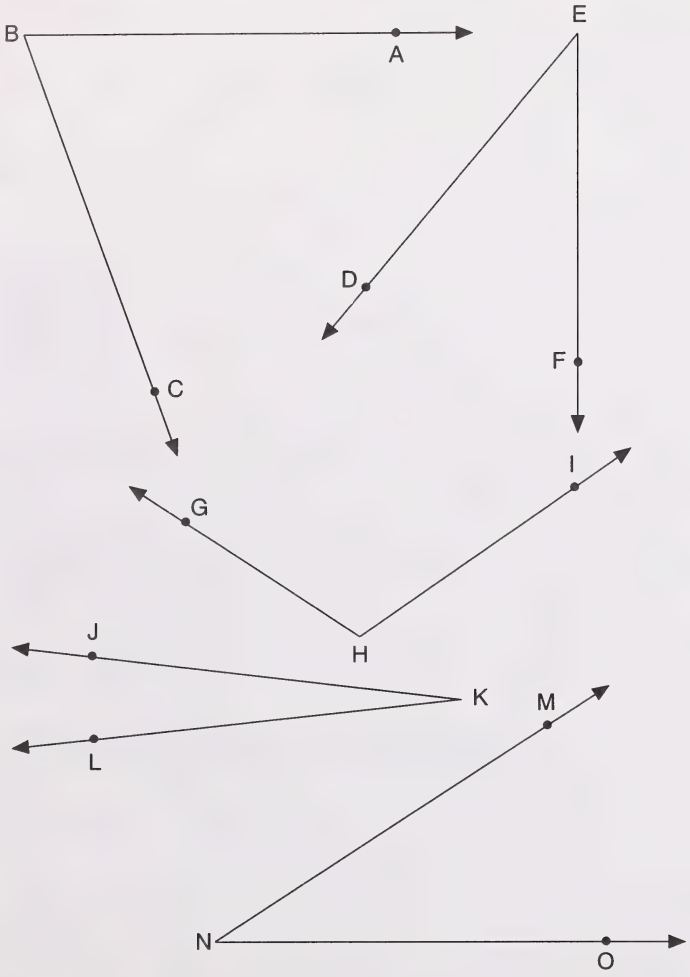
The angle is greater than 90°.

Practice Activity 1

1. Estimate the measure of each of the following line segments. Then measure the segments with a ruler.



2. Estimate the measures of each of the following angles. Then measure the angles with a protractor.



3. Sketch line segments with each of these measurements. Use a straightedge, but **do not** use a ruler.

   a. 5 cm **b.** 10 cm **c.** 15 cm

4. Measure the segments you sketched in Question 3 with a ruler. How accurate were your sketches?

5. Sketch angles with each of these measurements. Use a straightedge, but **do not** use a protractor.

   a. 45° **b.** 90° **c.** 135°

6. Measure the angles you sketched in Question 5 with a protractor. How accurate were your sketches?

Working Together

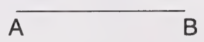
Congruent segments have the same measures.

Congruent angles have the same measures.

How can you copy a segment or an angle without using a ruler or a protractor? One way is to use tracing paper and a straightedge.

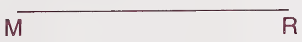
Example

Draw $\overline{MN}$ congruent to this segment.



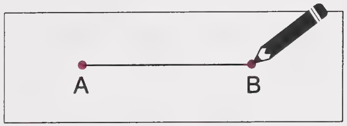
Solution

Step 1: Draw a segment $\overline{MR}$ on your paper.

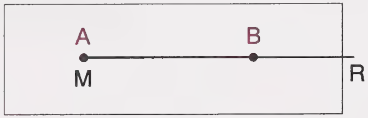


Ensure $\overline{MR}$ is longer than $\overline{AB}$, but do not measure.

Step 2: Cover $\overline{AB}$ with tracing paper and make a dot at each endpoint of $\overline{AB}$.

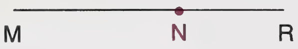


Step 3: Label the points on the tracing paper as A and B. Then place the tracing paper over $\overline{MR}$ on your paper. Ensure that A on the tracing paper matches with M on your paper and that B is on $\overline{MR}$.



Step 4: Press hard with your pencil on the point B on the tracing paper so that an impression is left on your paper.

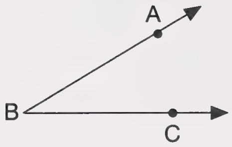
Step 5: Remove the tracing paper and indicate the point N on your paper.



$\overline{MN} \cong \overline{AB}$

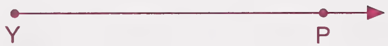
Example 2

Draw $\angle XYZ$ congruent to this angle.



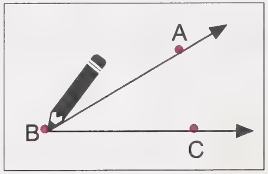
Solution

Step 1: Draw a ray, $\overrightarrow{YP}$, on your paper.

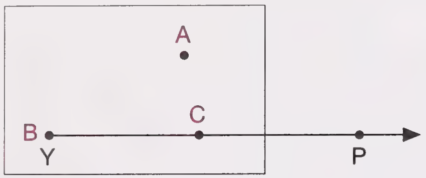


Ensure $\overline{YP}$ is longer than $\overline{BC}$, but do not measure.

Step 2: Cover $\angle ABC$ with tracing paper. Mark the vertex of $\angle ABC$ and a dot on each ray.

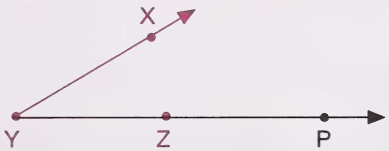


Step 3: Label the point at the vertex as B and the points on each ray as A and C. Then place the tracing paper over $\overrightarrow{YP}$ on your paper. Ensure that the point B on the tracing paper matches point Y on your paper and that C is on $\overrightarrow{YP}$.



Step 4: Press hard with your pencil at the points A and C on the tracing paper so that impressions are left on your paper.

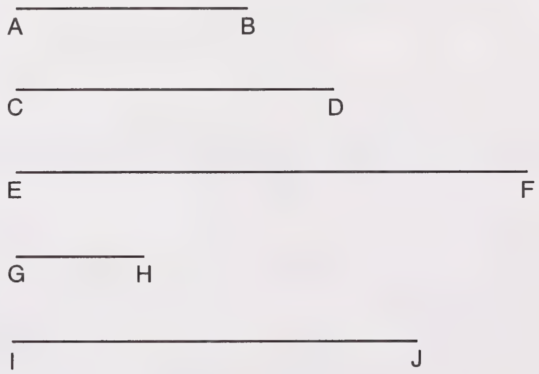
Step 5: Remove the tracing paper and use a straightedge and the impressions from X and Z to draw $\angle XYZ$. Label the angle.



$\angle XYZ \cong \angle ABC$

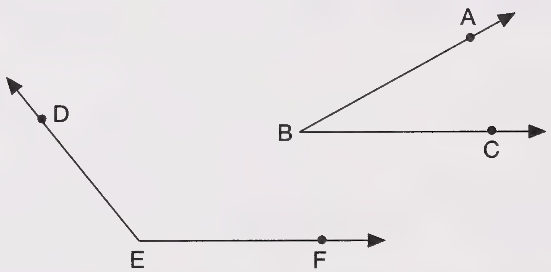
Practice Activity 2

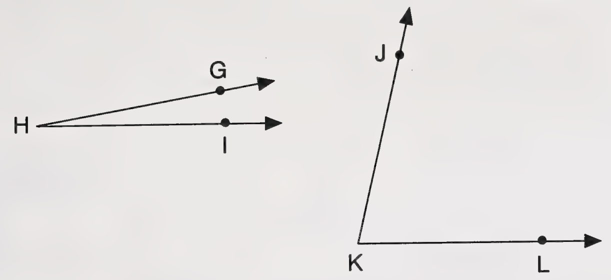
1. Use tracing paper to copy each of these segments.



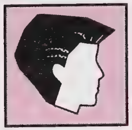
2. Check the accuracy of your copies in Question 1 with a ruler.

3. Use tracing paper to copy each of these angles.





4. Check the accuracy of your copies in Question 3 with a protractor.



Working Together

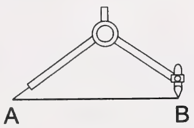
In Practice Activity 2 you copied segments and angles using tracing paper and a straightedge. A more accurate method of copying segments and angles is to use a compass and a straightedge. This is the method used by draftspeople and architects.

Example 1

Construct $\overline{XP}$ congruent to $\overline{AB}$.

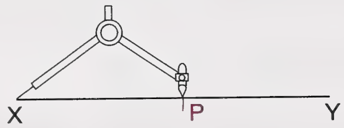
Solution

Step 1: Set the compass to the length of the segment.



Step 2: Use a straightedge to draw $\overline{XY}$ on your paper. Do not measure the segment, but make it longer than $\overline{AB}$.

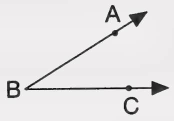
Step 3: Without changing the compass setting, place the point of the compass on X, and draw an arc cutting $\overline{XY}$ at P.



$\overline{XP} \cong \overline{AB}$

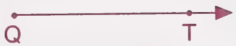
Example 2

Draw $\angle PQR$ congruent to $\angle ABC$.



Solution

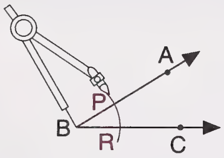
Step 1: Use a straightedge to draw $\overrightarrow{QT}$ on your paper.



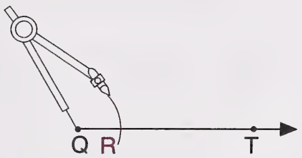
> Ensure that $\overline{QT}$ is longer than $\overline{BC}$, but do not measure.

Step 2: Place the compass at the vertex of the given angle, $\angle ABC$.

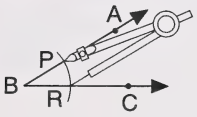
Then use the compass to draw an arc that intersects $\overrightarrow{BA}$ at P and $\overrightarrow{BC}$ at R.



Step 3: Without changing the compass setting, place the point of the compass on Q on your paper. Draw an arc that intersects $\overrightarrow{QT}$ at R.

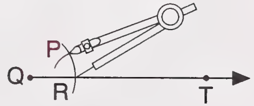


Step 4: Set the compass to the length of $\overline{PR}$, the distance from P to R.

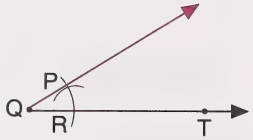


Measuring $\overline{PR}$ involves changing the setting of the compass.

Step 5: Keeping the same compass setting, place the point of the compass at R on your paper. Draw an arc to intersect the arc you drew previously at P.



Step 6: Draw $\overrightarrow{QP}$.



$\angle PQR \cong \angle ABC$

Video Activity

You may wish to watch the demonstration of copying an angle on the video *Geometric Constructions*.

Practice Activity 3

1. Draw five line segments. Copy these segments using only a compass and a straightedge.

2. Check the accuracy of your constructions in Question 1 using a ruler.

3. Draw five angles using only a compass and a straightedge.

4. Check the accuracy of your constructions in Question 3 using a protractor.

Working Together

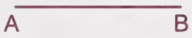
You can construct triangles by copying segments or angles.

Example 1

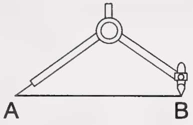
Construct an equilateral triangle.

Solution

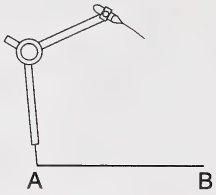
Step 1: Draw any segment and label it $\overline{AB}$.



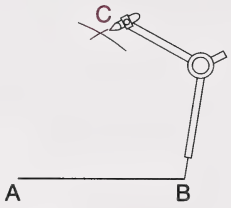
Step 2: Set the compass at the length of $\overline{AB}$.



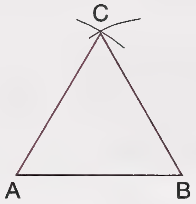
Step 3: Without changing the setting of the compass, place the point of the compass on A and make an arc above $\overline{AB}$.



Step 4: Keeping the same compass setting, place the point of the compass on B and draw an arc that intersects the first arc at C.



Step 5: Use a straightedge to draw $\overline{AC}$ and $\overline{BC}$.

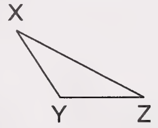


$\overline{AC} \cong \overline{AB} \cong \overline{BC}$

ΔABC is an equilateral triangle.

Example 2

Construct a triangle that is congruent to ΔXYZ.



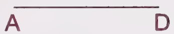
Solution

There are several ways to construct a triangle that is congruent to ΔXYZ.

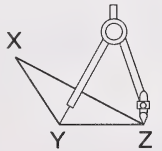
Method 1

You can copy $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.

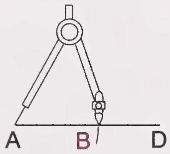
Step 1: Draw a segment on which to build the triangle.



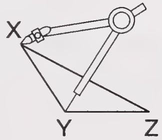
Step 2: Set the compass at the length of $\overline{YZ}$.



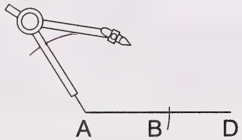
Step 3: Without changing the setting of the compass, place the point of the compass on A and make an arc cutting $\overline{AD}$ at B.



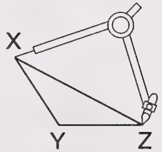
Step 4: Set the compass at the length of $\overline{XY}$.



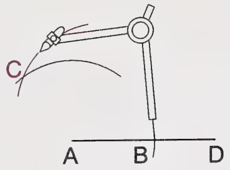
Step 5: Without changing the setting, place the point of the compass on A and make an arc above $\overline{AD}$.



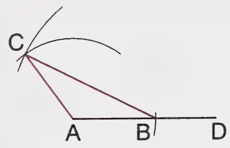
Step 6: Set the compass at the length of $\overline{XZ}$.



Step 7: Without changing the setting of the compass, place the point of the compass at B and make an arc above $\overline{AD}$ cutting the first arc at C.



Step 8: With a straightedge draw $\overline{CA}$ and $\overline{CB}$.



Method 2

You can copy $\overline{XY}$, $\angle XYZ$, and $\overline{XZ}$.

Method 3

You can copy $\angle XYZ$, $\overline{YZ}$, and $\angle YZX$.

Practice Activity 4

1. **a.** Draw an equilateral triangle with segments of this length.

   b. Draw an isoscles triangle with segments of these lengths.

   c. Draw a scalene triangle with segments of these lengths.

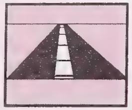
   d. Can you construct a scalene triangle with sides of these lengths? Why or why not?

2. **a.** Draw any triangle. Then copy the triangle.
   b. Test to see if the triangles are congruent using tracing paper.

3. Repeat Question 2 as many times as you wish.

Turn to the Appendix to check your answers.

BISECTING SEGMENTS AND ANGLES

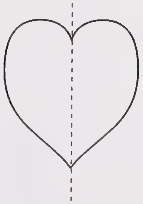


What Lies Ahead

In this section you will learn to bisect segments and angles. You will also learn to construct angles of 90°, 45°, 60°, and 30°.

Working Together

Earlier in this module you learned that a line of symmetry divides a figure into two congruent parts. In other words, it **bisects** the figure.



Bi- means "two".

The line of symmetry is also called a **bisector**.

Using a MIRA

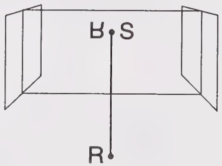
If you have a **MIRA**, a geometry tool made of plexiglass, you can use it to bisect a segment or an angle.

Example

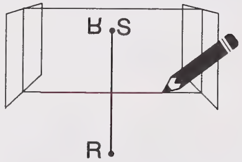
Bisect $\overline{SR}$.

Solution

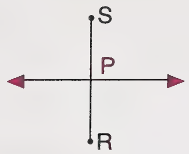
Step 1: Place your MIRA on $\overline{SR}$. Adjust the MIRA so that the image of R is reflected onto S.



Step 2: Use the MIRA to draw the bisector.



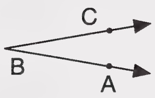
Step 3: Remove the MIRA and draw arrowheads on the bisector. Label the point of intersection as P.



$\overline{PS} \cong \overline{PR}$

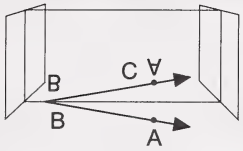
Example 2

Bisect $\angle ABC$.

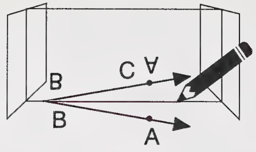


Solution

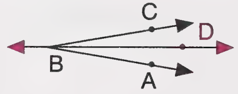
Step 1: Place your MIRA on B so that the image of $\overrightarrow{BA}$ is reflected on $\overrightarrow{BC}$.



Step 2: Use the MIRA to draw the bisector.



Step 3: Remove the MIRA and put arrowheads on the bisector. Label a second point on the bisector as D.



$\angle CBD \cong \angle DBA$

Practice Activity 1

If you have a MIRA, do this activity. If not, you may omit it.

1. Draw five line segments on your paper. Then use a MIRA to bisect each segment.

2. Check the accuracy of your construction in Question 1 with a ruler. Is each line segment divided into two congruent parts?

3. Draw five angles on your paper. Then use a MIRA to bisect each angle.

4. Check the accuracy of your constructions in Question 3 with a protractor. Is each angle divided into two congruent parts?

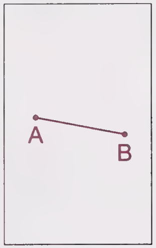
Working Together

Folding Paper

You can bisect a segment or an angle by folding paper.

Example 1

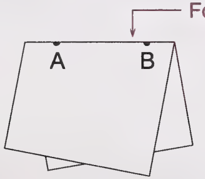
Draw a line segment on a sheet of tracing paper. Label the endpoints A and B.



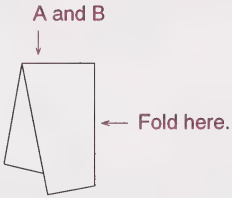
How can you bisect $\overline{AB}$?

Solution

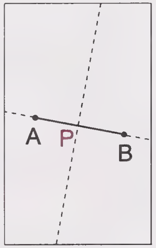
Step 1: Fold the paper along $\overline{AB}$.



Step 2: Fold the paper again so that point A falls on top of point B.



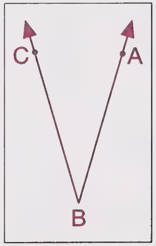
Step 3: Unfold the paper. Label the point of intersection as P.



$\overline{AP} \cong \overline{PB}$

Example 2

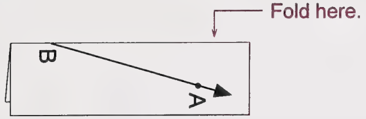
Draw an angle on a sheet of tracing paper.



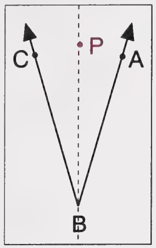
How can you bisect $\angle ABC$?

Solution

Step 1: Fold the paper so that $\overrightarrow{BA}$ falls on $\overrightarrow{BC}$.



Step 2: Unfold the paper. Label one point on the bisector as P.



$\angle CBP \cong \angle PBA$

Practice Activity 2

1. **a.** Draw a line segment on a piece of tracing paper. Bisect the line segment by folding the paper.

   b. Check the accuracy of your construction in Question 1.a. using a ruler.

   c. Measure each angle that is formed when the bisector intersects the line segment.

2. Repeat Question 1 as many times as you wish.

3. **a.** Draw an angle on a sheet of tracing paper. Bisect the angle by folding the paper.

   b. Check the accuracy of your construction in Question 3.a. with a protractor.

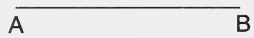
4. Repeat Question 3 as many times as you wish.

Working Together

You can bisect a segment or an angle with a compass and a straightedge.

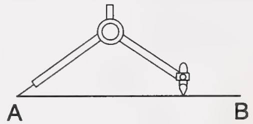
Example 1

Bisect $\overline{AB}$.

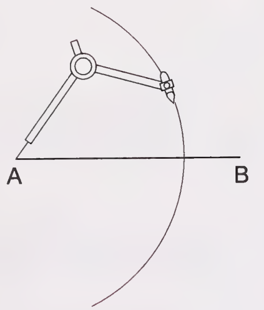


Solution

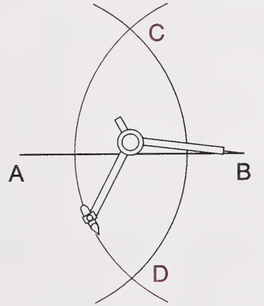
Step 1: Place the point of the compass on A. Open the compass to more than half the length of $\overline{AB}$.



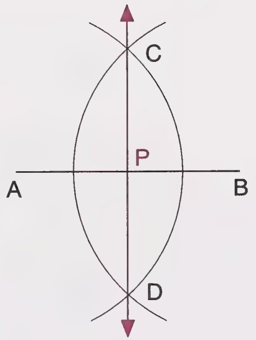
Step 2: Keep the compass setting the same and draw an arc cutting $\overline{AB}$. Ensure that the arc is about a semicircle.



Step 3: Without changing the setting on the compass, place the compass point at B. Then draw another arc that cuts the first arc at C and D.



Step 4: Draw a line through C and D with a straightedge. Label the point where the bisector cuts $\overline{AB}$ as P.

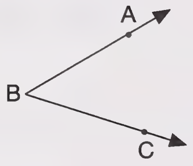


$\overline{AP} \cong \overline{PB}$

Note: The bisector intersects $\overline{AB}$ at right angles and is sometimes called the **perpendicular bisector**.

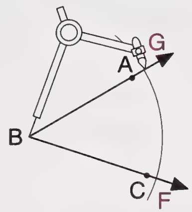
Example 2

Bisect $\angle ABC$.



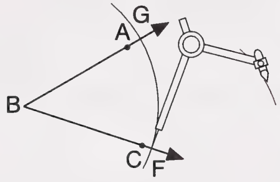
Solution

Step 1: Place the point of the compass on B. Draw an arc which intersects $\overrightarrow{BA}$ at G and intersects $\overrightarrow{BC}$ at F.



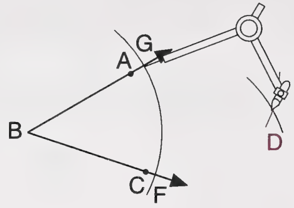
Note: Do not make $\overset{\frown}{FG}$ too close to the vertex of the angle. You may extend the rays of the angle if you wish.

Step 2: Place the point of the compass on F and draw an arc between the rays of the angle beyond $\overset{\frown}{FG}$.

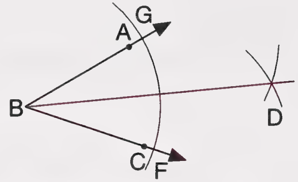


Make sure the arc extends at least halfway across the mouth of the angle.

Step 3: Without changing the setting of the compass, place the point of the compass on G and draw an arc that intersects the arc drawn in Step 2 at D.



Step 4: Use a straightedge to draw the bisector $\overline{BD}$.



$\angle ABD \cong \angle DBC$

Video Activity

You may wish to view the demonstration of bisecting segments and angles on the video *Geometric Constructions*.

Practice Activity 3

1. **a.** Draw a segment on your paper. Then bisect the segment using only a compass and a straightedge.

   b. Check the accuracy of your construction in Question 1.a. with a ruler.

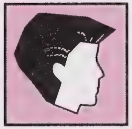
   c. Measure each angle formed where the bisector meets the segment.

2. Repeat Question 1 as many times as you wish.

3. **a.** Draw an angle on your paper. Then bisect the angle using only a compass and a straightedge.

   b. Check the accuracy of your construction in Question 3.a. with a protractor.

4. Repeat Question 3 as many times as you wish.



Working Together

Now that you have learned to copy segments and bisect angles, you can construct many angles using only a compass and a straightedge.

Example 1

In order to construct a right angle, you can bisect a straight angle.

Example 2

In order to construct a 45° angle, you can bisect a straight angle and then bisect the right angle that is formed.

Example 3

To construct a 60° angle, you can construct an equilateral triangle.

Example 4

To construct a 30° angle, you can construct an equilateral triangle, and then bisect one of its angles.

Practice Activity 4

1. Construct each of the following angles using only a compass and a straightedge.

   a. 90° **b.** 45° **c.** 60° **d.** 30°

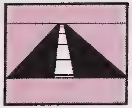
2. Construct each of the following angles using a compass and a straightedge.

   a. 135° **b.** 105° **c.** 75°

3. Check the accuracy of your constructions in Questions 1 and 2 with a protractor.

Turn to the Appendix to check your answers.

CONSTRUCTING PERPENDICULAR LINES



What Lies Ahead

In this section you will learn to construct perpendicular lines.

Working Together

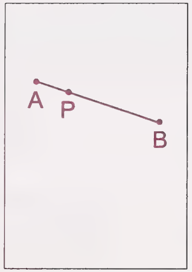
In the previous section you learned to construct a perpendicular bisector of a segment. In this section you will learn to construct other perpendicular lines.

Folding Paper

You can draw these perpendicular lines by folding paper.

Example 1

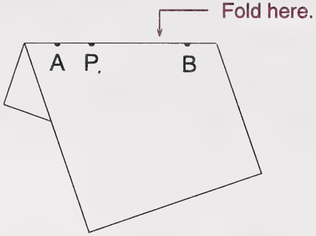
Draw a line segment, $\overline{AB}$, on a piece of tracing paper. Place a point, P, anywhere on the segment.



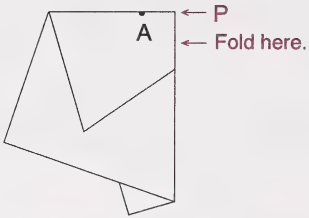
How can you draw a line perpendicular to $\overline{AB}$ through P?

Solution

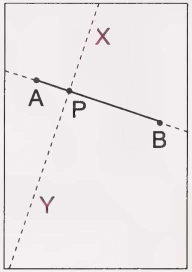
Step 1: Fold the paper along $\overline{AB}$.



Step 2: Fold the paper at right angles to $\overline{AB}$ through P.



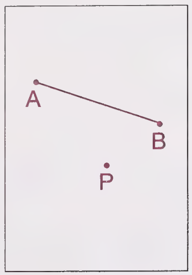
Step 3: Unfold the paper and label the perpendicular line $\overleftrightarrow{XY}$.



$\overleftrightarrow{XY} \perp \overline{AB}$ and passes through P.

Example 2

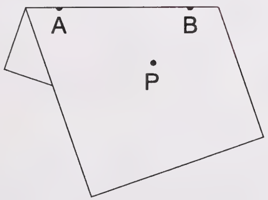
Draw a line segment, $\overline{AB}$, on a piece of tracing paper. Place a point, P, anywhere above or below $\overline{AB}$.



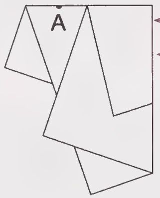
How can you draw a line perpendicular to $\overline{AB}$ through P?

Solution

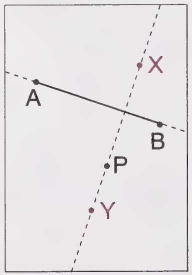
Step 1: Fold the paper along $\overline{AB}$.



Step 2: Fold the paper at right angles along $\overline{AB}$ through P.



Step 3: Unfold the paper and label the perpendicular line $\overleftrightarrow{XY}$.



$\overleftrightarrow{XY} \perp \overline{AB}$ and passes through P.

Practice Activity 1

1. Draw a line segment on a piece of tracing paper. Put a point anywhere on the line segment.

   a. Construct a line perpendicular to the line segment through the point by folding the paper.

b. Check the accuracy of your construction in Question 1.a. with a protractor.

2. Repeat Question 1 as many times as you wish.

3. Draw a line segment on a piece of tracing paper. Put a point anywhere above or below the line segment.

   a. Construct a perpendicular line to the line segment through the point by folding.

   b. Check the accuracy of your construction in Question 3.a. with a protractor.

4. Repeat Question 3 as often as you wish.

Working Together

You can construct perpendicular lines with only a straightedge and a compass.

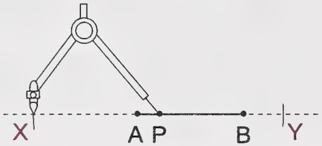
Example 1

Construct a line perpendicular to $\overline{AB}$ through P.

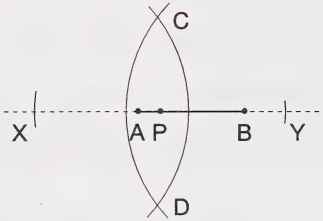
Solution

Step 1: Extend $\overline{AB}$.

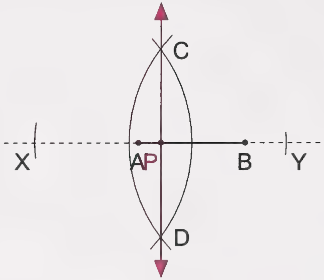
Step 2: Using any setting, place the point of the compass at P and draw arcs cutting $\overline{AB}$ on each side of P at X and Y.



Step 3: Bisect $\overline{XY}$.



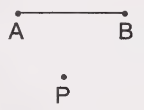
Step 4: Draw the perpendicular bisector of $\overline{XY}$. It will pass through P.



$\overleftrightarrow{CD} \perp \overline{AB}$ and passes through P.

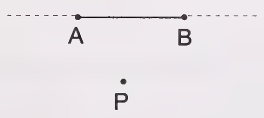
Example 2

Construct a line perpendicular to $\overline{AB}$ through P.

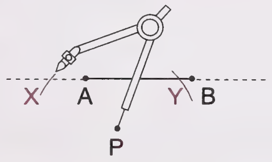


Solution

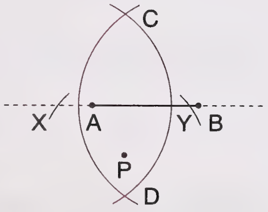
Step 1: Extend $\overline{AB}$.



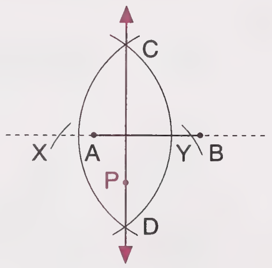
Step 2: Using any setting of the compass, place the point of the compass at P and draw arcs cutting $\overline{AB}$ at X and Y.



Step 3: Bisect $\overline{XY}$.



Step 4: Draw the perpendicular bisector of $\overline{XY}$.



$\overleftrightarrow{CD} \perp \overline{AB}$ and passes through P.

Practice Activity 2

1. **a.** Draw a line segment with a point anywhere on the segment. Construct a perpendicular to the line segment through the point. Use only a compass and a sraightedge.

   b. Check the accuracy of your construction in Question 1.a. with a protractor.

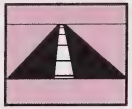
2. Repeat Question 1 as often as you wish.

3. **a.** Draw a line segment with a point somewhere above or below the segment. Construct a perpendicular to the segment through the point. Use only a compass and a straightedge.

   b. Check the accuracy of your construction in Question 3.a. with a protractor.

4. Repeat Question 3 as often as you wish.

PERIMETER AND CIRCUMFERENCE



What Lies Ahead

In this section you will develop and use formulas to calculate the perimeter of a polygon and the circumference of a circle indirectly. You will also solve problems involving perimeter or circumference.

Working Together

Perimeter is the distance around a figure. Perimeter is a measurement of length.

Finding the perimeter of an object is a useful skill for everyday life. For example, to find how much wood you need to fence your property, you need to find the perimeter.

The perimeter of a circle is called the **circumference**. Circumference is a measurement of length.

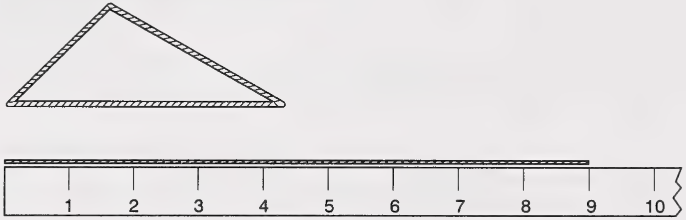
Finding the circumference of a circle is also a useful skill. For example, to find the amount of lace required around the bottom of a skirt, you need to find the circumference of the skirt.

You can find the perimeter of a polygon or the circumference of a circle directly.

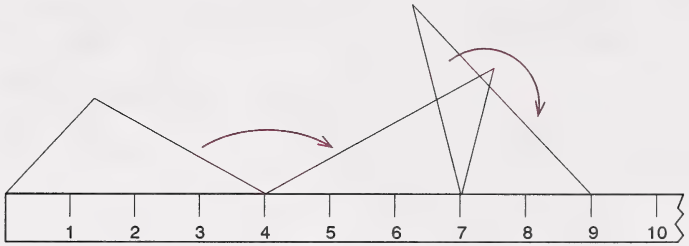
Example

You can find the perimeter of triangle directly in several ways.

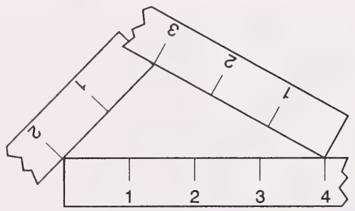
- You can use string to help you find the perimeter of the triangle. Then measure the string.



- You can "roll" the triangle along a metre stick.



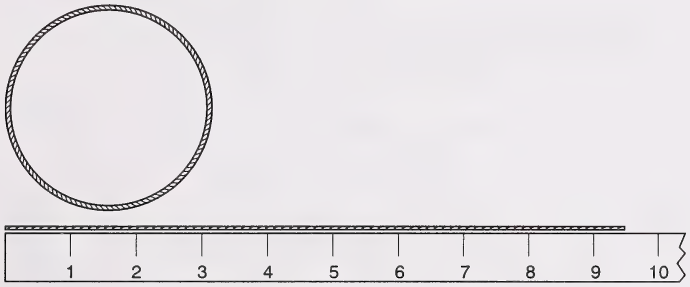
- You can measure each side of the triangle and find the sum.



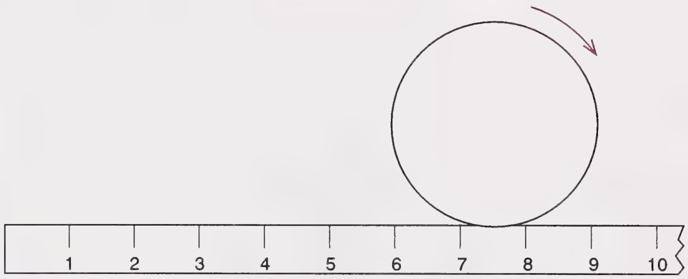
Example 2

You can find the circumference of a circle directly in several ways.

- You can use string to help you find the circumference. Then measure the string.



- You can "roll" the circle along a metre stick.



These methods, however, are tedious and awkward.

You can use formulas as an indirect method of finding the perimeter of a polygon or the circumference of a circle.

Perimeter of Rectangles and Parallelograms

The opposite sides of rectangles and parallelograms are congruent. This property helps you to calculate the perimeter when you are given the lengths of only two sides.

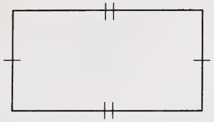
You can find the perimeter (P) of any rectangle or parallelogram by doubling the length (ℓ) and the width (w) and then adding the products.

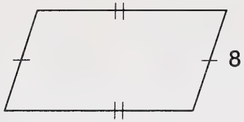
This rule can be expressed by this formula.

$$P = 2\ell + 2w$$

You can use this formula to find the perimeter of other rectangles and parallelograms without measuring all four sides.

Example





$$\begin{aligned} P &= 2\ell + 2w \\ &= 2 \times 15 + 2 \times 8 \\ &= 30 + 16 \\ &= 46 \end{aligned}$$

The perimeter of both the rectangle and the parallelogram is 46 cm.

Perimeter of a Regular Polygon

A **regular polygon** is a polygon whose sides are line segments of equal length. This property helps you calculate the perimeter when you are given the length of one side.

You can find the perimeter (P) of any regular polygon by multiplying the number of equal sides (n) by the length of one side (s).

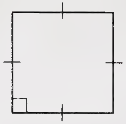
This rule can be expressed by this formula.

$$P = ns$$

You can use this formula to find the perimeter of any regular figure.

Example 1

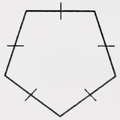
$$\begin{aligned} P &= n \times s \\ &= 4 \times 8.6 \\ &= 34.4 \end{aligned}$$



The perimeter of the square is 34.4 m.

Example 2

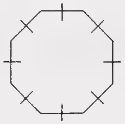
$$\begin{aligned} P &= n \times s \\ &= 5 \times 9.8 \\ &= 49 \end{aligned}$$



The perimeter of the regular pentagon is 49 cm.

Example 3

$$\begin{aligned} P &= n \times s \\ &= 8 \times 1.5 \\ &= 12 \end{aligned}$$



The perimeter of the regular octagon is 12 m^2.

Circumference of a Circle

The ratio of the circumference of a circle to the diameter of the circle is the same for all circles regardless of their size. This ratio is a constant value that has an infinite number of decimal places.

$$3.141592\cdots \doteq 3.14$$

This ratio is represented by the Greek letter π (pi).

This property helps you to calculate the circumference when you are given the diameter of the circle.

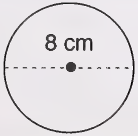
$$\frac{C}{d} = \pi$$

So, $C = \pi d$.

You can use this formula to find the circumference of any circle.

Example 1

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14 \times 8 \\ &\doteq 25.12 \end{aligned}$$



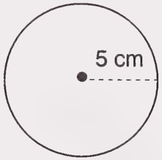
The circumference of the circle is about 25.12 cm.

Note: Some calculators have a key for π. Does your calculator have this key? If so, you may use it to perform calculations.

Example 2

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14 \times 10 \\ &\doteq 31.4 \end{aligned}$$

$$\begin{aligned} d &= 2r \\ &= 2 \times 5 \\ &= 10 \end{aligned}$$

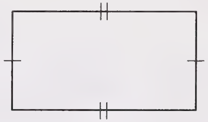


The circumference of the circle is about 31.4 cm.

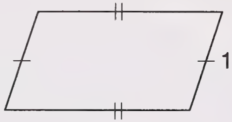
Practice Activity 1

1. Calculate the perimeter of each polygon.

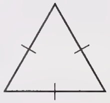
a.



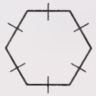
b.



c.

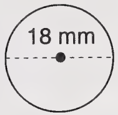


d.



2. Calculate the circumference of each of these circles. Round your answers to the nearest tenth.

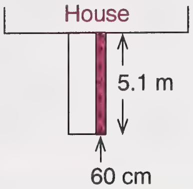
a.



b.

3. The Pentagon in Washington, D.C. is so named because of its shape. Each of its outer walls is 302 m long. Find the minimum distance (in kilometres) that you would travel while walking around the outside of the Pentagon.

4. There is a rectangular flower bed beside the walkway to a house. If the flower bed is 60 cm wide and 5.1 m long, what is the perimeter of the flower bed?



Turn to the Appendix to check your answers.

Working Together

Problems may involve the perimeter of a polygon or the circumference of a circle.

You may use any method to solve these problems.

Example

The difference between the width and the length of the floor of a barn is 6 m. If the perimeter of the floor of the barn is 76 m, what are the dimensions of the floor?

There are several methods to solve this problem.

Method 1: Guessing, Checking, and Revising

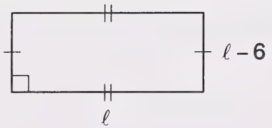
	Length	Width	Test
Guess 1	18	12	$2 \times 18 + 2 \times 12 \neq 76$
Guess 2	20	14	$2 \times 20 + 2 \times 14 \neq 76$
Guess 3	22	16	$2 \times 22 + 2 \times 16 = 76$

The floor of the barn is 22 m by 16 m.

Method 2: Using Algebra

Let ℓ be the length of the floor.
Let $\ell - 6$ be the width of the floor.

$$
\begin{aligned}
2\ell + 2(\ell - 6) &= 76 \\
2\ell + 2\ell - 12 &= 76 \\
4\ell \cancel{-12} &= 76 \\
\cancel{+12} \quad & \;\; +12 \\
\hline
4\ell &= 88 \\
\ell &= 22
\end{aligned}
$$



If $\ell = 22$, then

$$
\begin{aligned}
\ell - 6 &= 22 - 6 \\
&= 16
\end{aligned}
$$

The floor of the barn is 22 m by 16 m.

Practice Activity 2

1. Do the puzzle "What's the Quickest Way for an Ant to Go From the Ground to the Tree Trunk?"[1]

2. A farmer wishes to put a fence around his pasture with three strands of barbed wire. The field is a square which is 820 m on a side. What length of barbed wire will he have to purchase?

[1] 1989 Creative Publications for exerpt from *Algebra With Pizzazz*.

3. Clinton has just completed a nonstop bicycle trip of 6 km. The diameter of each wheel of his bicycle is 70 cm.

   a. How far does each wheel travel in one turn?

   b. How many turns did each wheel make in the nonstop trip of 6 km?

4. The diameter of a basketball is 24.5 cm. The diameter of a basketball hoop is 45 cm. (You may wish to draw a diagram to help you answer this problem.)

   a. The circumference of the hoop is how much greater than the circumference of the basketball?

   b. If the ball goes through the centre of the hoop, find the distance between the ball and the hoop.

5. The diameter of the earth at the equator is approximately 12 750 km.

   a. The circumference of the earth is about how many kilometres?

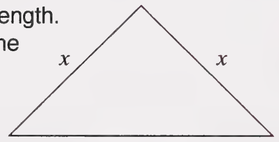
   b. If a satellite is orbiting the earth 36 000 km above the earth's surface, how far does it travel in completing one orbit?

Turn to the Appendix to check your answers.

WHAT'S THE QUICKEST WAY FOR AN ANT TO GO FROM THE GROUND TO THE TREE TRUNK?

Solve each ohe following problems. Find the solution in the answer column and notice the letter next to it. Write this letter in each box that contains the number of that problem.

1. The length of a rectangle is three times the width. The perimeter is 96 cm. Find the width and length.
2. The length of a rectangle is 5 m greater than the width. The perimeter is 150 m. Find the width and length.
3. The width of a rectangle is 12 cm less than the length. The perimeter is 156 cm. Find the width and length.
4. The length of a rectangle is 2 cm less than seven times the width. The perimeter is 60 cm. Find the width and length.
5. The perimeter of a triangle is 76 cm. Side a of the triangle is twice as long as side b. Side c is 1 cm longer than side a. Find the length of each side.
6. The first side of a triangle is 8 m shorter than the second side. The third side is four times as long as the first side. The perimeter is 26 m. Find the length of each side.
7. A triangular sail has a perimeter of 25 m. Side a is 2 m shorter than twice side b, and side c is 3 m longer than side b. Find the length of each side.
8. The triangle shown at the right is isosceles. That is, it has two sides of equal length. The third side is 30 m shorter than twice the length of each congruent side. The perimeter is 570 m. Find the length of each side.



- (B) 37 m, 42 m
- (O) 3 m, 11 m, 12 m
- (A) 33 cm, 45 cm
- (N) 8 m, 5 m, 11 m
- (E) 12 cm, 36 cm
- (S) 4 cm, 26 cm
- (U) 31 cm, 43 cm
- (M) 140 cm, 140 m, 250 m
- (H) 35 m, 40 m
- (C) 5 cm, 33 cm
- (K) 30 cm, 15 cm, 31 cm
- (T) 150 m, 150 m, 270 m
- (L) 32 cm, 16 cm, 31 cm
- (R) 10 m, 6 m, 9 m

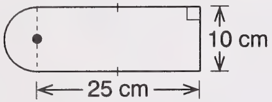
8	3	5	1	8	2	1	4	2	6	7	8	1	4	8	7	6	6	8

Working Together

The perimeter of a complex figure can be calculated by breaking it down into parts.

Example

Find the perimeter of this figure.

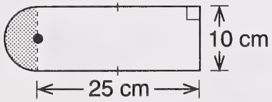


Solution

The perimeter of this figure is made up of a semicircle, two segments of 25 cm, and one segment of 10 cm. The diameter of the semicircle is 10 cm.

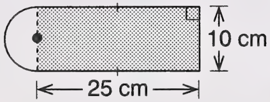
$$\begin{aligned} C &= \pi d \\ &\doteq 3.14 \times 10 \\ &\doteq 31.4 \end{aligned}$$

$\frac{1}{2}$ of $31.4 = 15.7$



The semicircle has a perimeter of about 15.7 cm.

$$\begin{aligned} P &= 15.7 + 2 \times 25 + 10 \\ &= 15.7 + 50 + 10 \\ &= 75.7 \end{aligned}$$

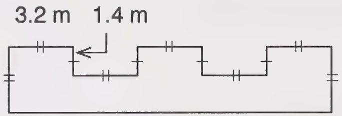


The perimeter of the figure is about 75.7 cm.

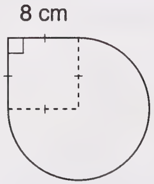
Practice Activity 3

Calculate the perimeter of the following figures.

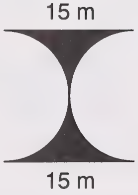
1.



2.

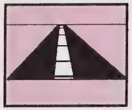


3.



Turn to the Appendix to check your answers.

AREA



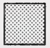
What Lies Ahead

In this section you will develop and use formulas to calculate the area of polygons and circles indirectly.

Working Together

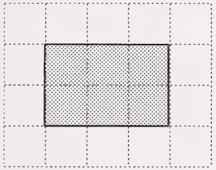
Area is used in many everyday situations, such as buying carpet for your living room or sod for your lawn.

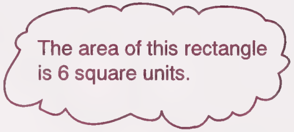
Area is a measurement of the surface that a figure contains. Square units are used to measure area.



Square unit

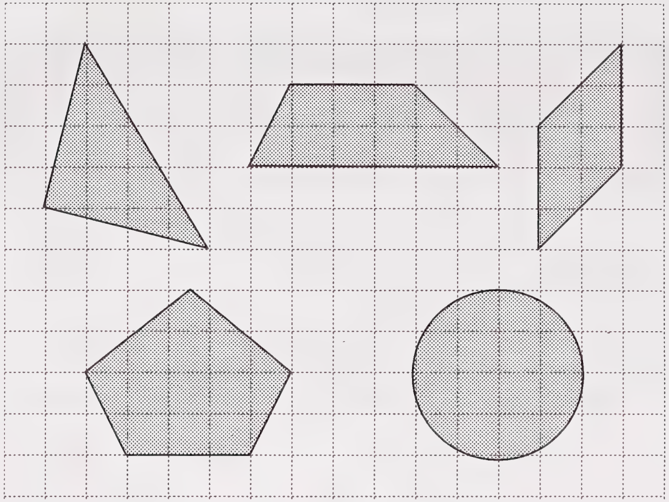
You can find the area of rectangles directly by counting the square units.





However, it is awkward and time consuming to count the square units.

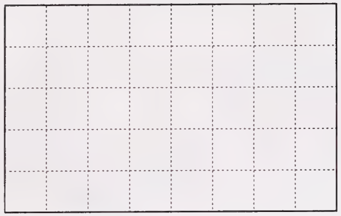
It is also difficult to find the area of these figures directly by counting because they are not made up of whole squares.



You can calculate the area of polygons and circles indirectly by using formulas.

Area of a Rectangle

Suppose you want to find the area of this rectangle.



Height

Base

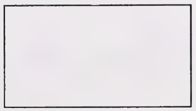
You do not need to count all the square units to find the area of a rectangle. You can simply count the number of square units along the base and multiply by the number of square units along the height. This rule can be expressed with a formula.

Area of a rectangle = base × height   or $A = bh$

You can use this formula to find the area of any rectangle indirectly.

Example

$$\begin{aligned} A &= b \times h \\ &= 10 \times 6 \\ &= 60 \end{aligned}$$



6 cm

10 cm

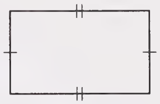
The area is 60 cm^2.

60 cm^2 is read as "60 square centimetres".

Practice Activity 1

1. Calculate the area of each of the following rectangles.

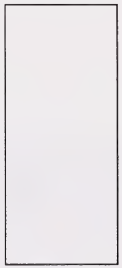
a.



4 cm

6.8 cm

b.



40 mm

17 mm

2. Jennifer would like to install a rectangular concrete pad with a length of 12 m and a width of 5.5 m in front of her house. What would the area of the pad be?

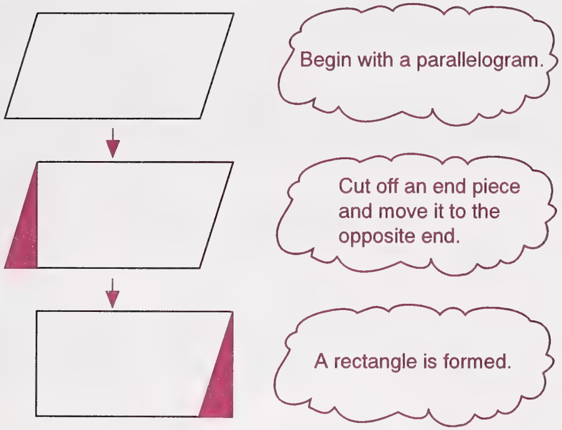
3. Karen wishes to carpet her rectangular living room. If the room is 3.4 m by 7.5 m, and the carpet she prefers is \$32.95/m^2, how much will the carpet cost?

Turn to the Appendix to check your answers.

Area of a Parallelogram

The two congruent opposite sides of a parallelogram are the **bases**. The perpendicular distance from a vertex to the opposite base is called the **height**.

To find the area of the parallelogram, you can cut off one of the triangular end pieces and slide it to the opposite end of the parallelogram. The new figure that is formed is a rectangle.



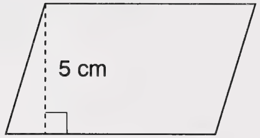
The base and height of the rectangle formed is the same as the base and height of the parallelogram. Therefore, the formula for the area of a rectangle can be used to find the area of a paralellogram.

Area of parallelogram = base × height   or $A = b \times h$

You can use this formula to find the area of any parallelogram indirectly.

Example

$A = b \times h$
$= 8 \times 5$
$= 40$

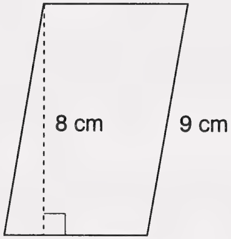


The area of the parallelogram is 40 cm^2.

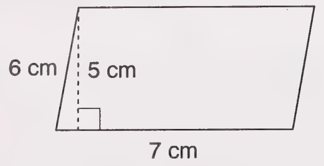
Practice Activity 2

1. Calculate the area of each parallelogram.

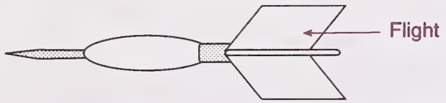
   a.



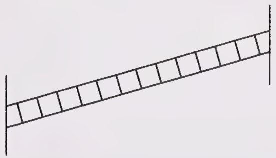
b.



2. The plastic flight on a dart is in the shape of a parallelogram. Its base is 3.2 cm and its height is 1.7 cm. What is the area of one flight.



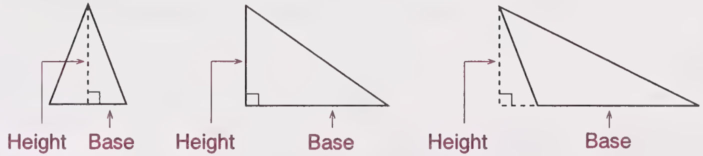
3. The sidewalk between two buildings is a parallelogram. Find its area. Its base is 16 m and its height is 2.5 m.



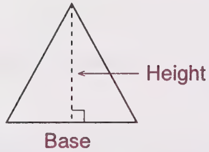
Turn to the Appendix to check your answers.

Area of a Triangle

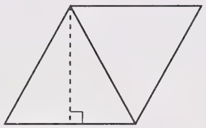
Triangles have a base and a height. The height is the perpendicular distance from a vertex to the opposite base or the extension of the base.



Suppose you want to find the area of this triangle.



Take a congruent triangle, turn it upside down and place it beside the triangle.



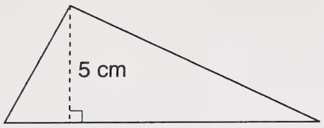
A parallelogram is formed. It has the same base and height as the triangle, but twice the area. Therefore, the area of the triangle can be found by finding one half of the area of the parallelogram.

$$\text{Area of triangle} = \frac{\text{base} \times \text{height}}{2} \quad \text{or} \quad A = \frac{b \times h}{2}$$

You can use this formula to find the area of any triangle indirectly.

Example 1

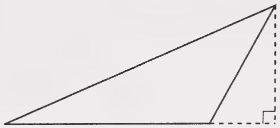
$$A = \frac{b \times h}{2}$$
$$= \frac{16 \times 5}{2}$$
$$= 40$$



The area of the triangle is 40 cm^2.

Example 2

$$A = \frac{b \times h}{2}$$
$$= \frac{7 \times 4.5}{2}$$
$$= 15.75$$



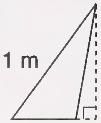
The area of the triangle is 15.75 cm^2.

Practice Activity 3

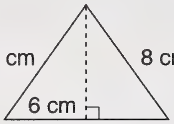
1. Use a formula to calculate the area of each triangle.

a.

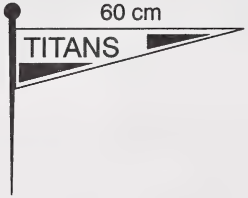
b.



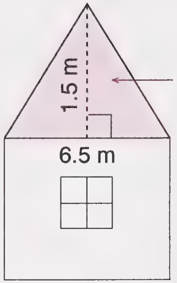
c.



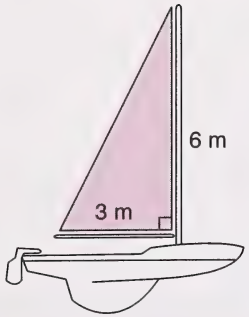
2. What is the area of this pennant?



3. What is the area of the gable of this house?



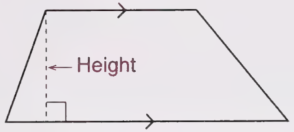
4. What is the area of the sail of this boat?



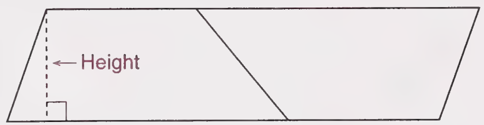
Turn to the Appendix to check your answers.

Area of a Trapezoid

The parallel sides of a trapezoid are called bases. The height of a trapezoid is the perpendicular distance from a vertex to the opposite base.



Suppose you want to find the area of the trapezoid. Take a congruent trapezoid, turn it upside down, and place it beside the trapezoid.



A parallelogram is formed. The base of this parallogram is the sum of base one and base two. So, the area of the parallelogram formed can be found by this formula.

$$\text{Area of parallelogram} = (\text{base one} + \text{base two}) \times \text{height}$$

or

$$A = (b_1 + b_2) \times h$$

The area of the trapezoid is half the area of the parallelogram. Therefore, the area can be found by this formula.

$$\text{Area of trapezoid} = \frac{(\text{base one} + \text{base two}) \times \text{height}}{2}$$

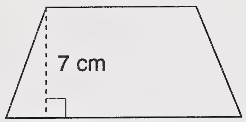
or

$$A = \frac{(b_1 + b_2) \times h}{2}$$

You can use this formula to find the area of any trapezoid indirectly.

Example

$$A = \frac{(b_1 + b_2) \times h}{2}$$
$$= \frac{(8+18) \times 7}{2}$$
$$= \frac{26 \times 7}{2}$$
$$= 91$$

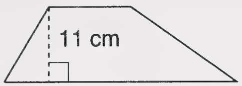


The area of the trapezoid is 91 cm^2.

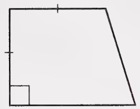
Practice Activity 4

1. Calculate the area of each of the following trapezoids.

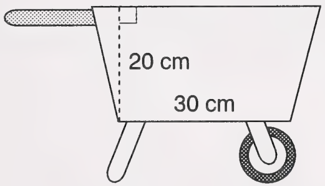
a.



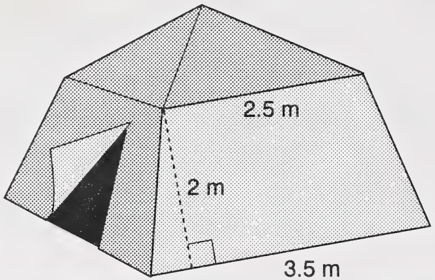
b.



2. Find the area of this side of a wheel barrow.



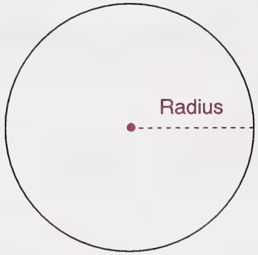
3. Find the area of this side of the tent.



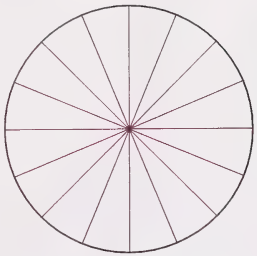
Turn to the Appendix to check your answers.

Area of a Circle

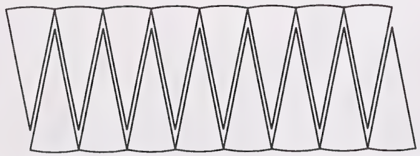
Suppose you want to find the area of this circle.



You can divide the circle into 16 congruent parts.



You can cut out and reassemble the pieces like this.



A shape which resembles a parallelogram is formed. The base of the parallelogram is one-half the circumference of the circle. The height of the parallelogram is equal to the radius of the circle. So, the area of the parallelogram can be found by this formula.

Area of parallelogram = $\frac{1}{2}$ circumference × diameter

or

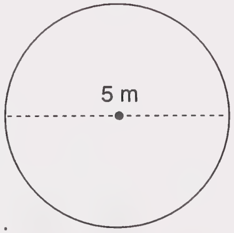
$$A = \frac{1}{2} \times (\pi \times d) \times r$$

$$= \frac{1}{\cancel{2}_{1}} \times \pi \times \cancel{2}^{1} r \times r \qquad \boxed{d = 2r}$$

$$= \pi r^2$$

The circle has the same area as the parallelogram so you can use this formula to find the area of any circle indirectly.

Example



$A = \pi \times r^2$

$A = 3.14 \times 5 \times 5$

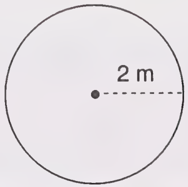
$A = 78.5$

The area of the circle is 78.5 m^2.

Practice Activity 5

1. Calculate the area of each circle to the nearest tenth.

   a.

   b.

2. Munir ties his horse to a post in the field with a 10-m rope. If the rope does not become tangled, what is the total area of grass the horse can eat?

3. The rotary sprinkler head on an irrigation system can spray water a distance of 25 m. What area of the field can be watered by this sprinkler?

4. A pizza restaurant has a lunch special where you can either order two 15-cm diameter pizzas or one 25-cm diameter pizza for the same price. Which deal offers the greatest area of pizza?

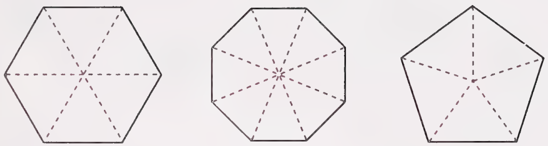
Turn to the Appendix to check your answers.

Working Together

Area of a Regular Polygon

In Section 3 you learned that regular polygons can be divided into congruent triangles.

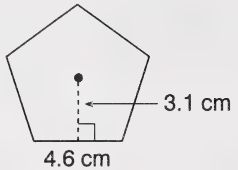
Example



This property helps you find the area of a regular polygon.

Example

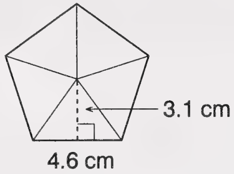
Find the area of this regular pentagon.



Solution

Divide the pentagon into five congruent triangles and find the area of one triangle.

$$A = \frac{bh}{2}$$
$$= \frac{4.6 \times 3.1}{2}$$
$$= 6.9$$



The area of one triangle is 6.9 cm^2.

Find the area of the regular polygon.

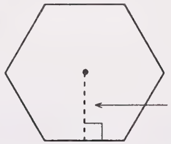
$5 \times 6.9 = 34.5$

The area of the regular polygon is 34.5 cm^2.

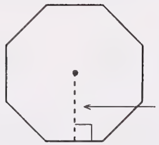
Practice Activity 6

Find the areas of the following regular polygons.

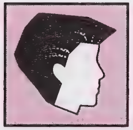
1.



2.

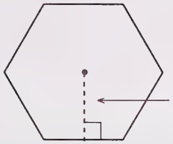


Turn to the Appendix to check your answers.

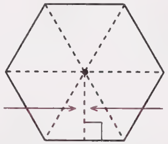


Working Together

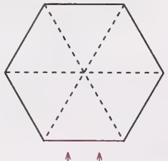
The line segment from the centre of a regular polygon perpendicular to one of its sides is the **apothem**.



The apothem and the altitude of the triangle formed are the same segment.



Also the base of the triangle is the same segment as the side of the regular hexagon.



Area of the hexagon = 6 × the area of the triangle

$$= 6 \times \frac{\text{height} \times \text{height}}{2}$$

$$= 6 \times \frac{\text{side} \times \text{apothem}}{2}$$

or

$$A = \frac{6 \times s \times a}{2}$$

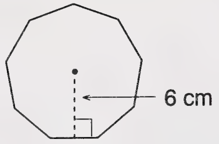
The area of any regular polygon can be found by this formula where n is the number of sides and the number of triangles formed.

$$A = \frac{n \times s \times a}{2}$$

Example

Find the area of this regular nonagon.

$$A = \frac{n \times s \times a}{2}$$
$$= \frac{9 \times 4 \times 6}{2}$$
$$= 108$$

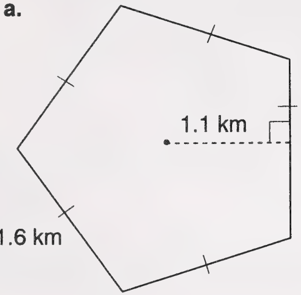


The area of the regular nonagon is 108 cm^2.

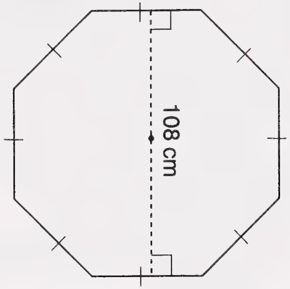
Practice Activity 7

1. Calculate the area of each of the following regular polygons.

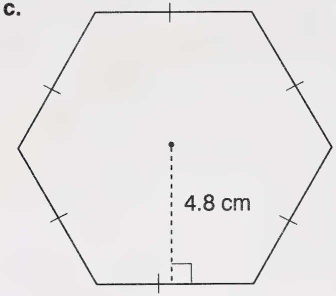
a.



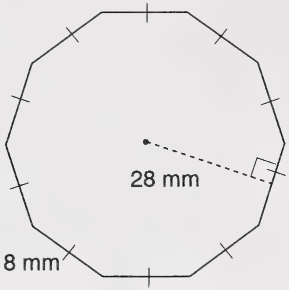
b.



c.



d.



2.

A stop sign is shaped like a reguar octagon, with an apothem of 20 cm and a base of 16.5 cm. What is the area of the sign?

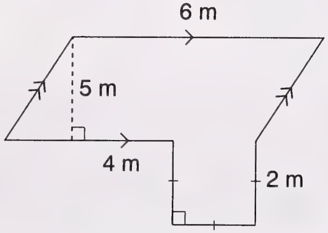
Turn to the Appendix to check your answers.

Working Together

Areas of complex figures can be calculated by breaking them into familiar figures and adding or subtracting areas.

Example 1

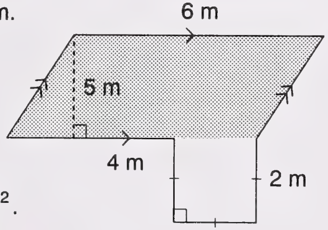
Find the area of this figure.



Solution

The area of the figure is equal to the area of the parallelogram and the area of the square.

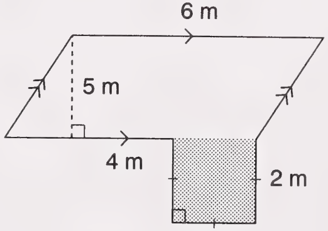
Calculate the area of the parallelogram.



$$
\begin{aligned}
A &= b \times h \\
&= 6 \times 5 \\
&= 30
\end{aligned}
$$

The area of the parallelogram is 30 m^2.

Calculate the area of the square.



$$
\begin{aligned}
A &= b \times h \\
&= 2 \times 2 \\
&= 4
\end{aligned}
$$

The area of the square is 4 m^2.

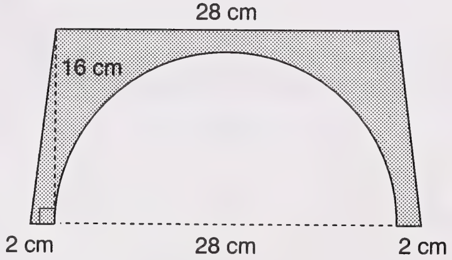
Calculate the area of the figure.

$30 + 4 = 34$

The area of the figure is 34 m^2.

Example

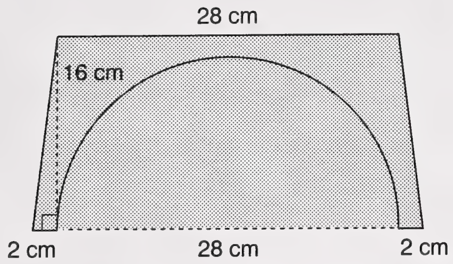
Find the area of this figure.



Solution

The area of the figure is equal to the area of the trapezoid minus the area of the semicircle.

Find the area of the trapezoid.



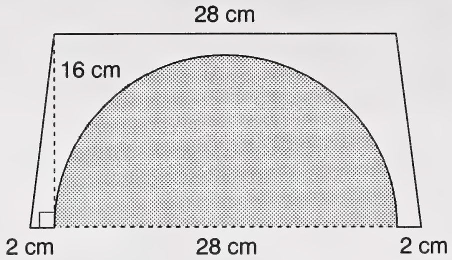
$$A = \frac{h \times (b_1 + b_2)}{2}$$
$$= 16 \times (28 + 32)$$
$$= 16 \times 60$$
$$= 960$$

$b_1 = 2 + 28 + 2$
$= 32$

The area of the trapezoid is 960 cm^2.

Find the area of the semicircle.

$$A = \frac{\pi r^2}{2}$$
$$\doteq \frac{3.14 \times 14^2}{2}$$
$$\doteq \frac{3.14 \times 196}{2}$$
$$\doteq \frac{615.44}{2}$$
$$\doteq 307.72$$



The area of the semicircle is about 307.72 cm^2.

Calculate the area of the figure.

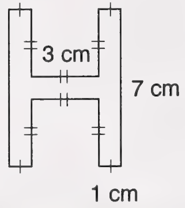
$960 - 307.72 = 652.28$

The area of the figure is about 652.28 cm^2.

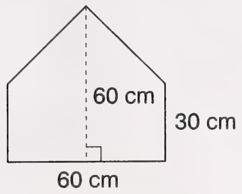
Practice Activity 8

1. Calculate the area of each of the following figures.

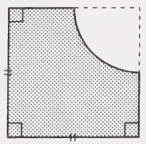
a.



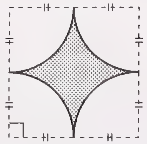
b.



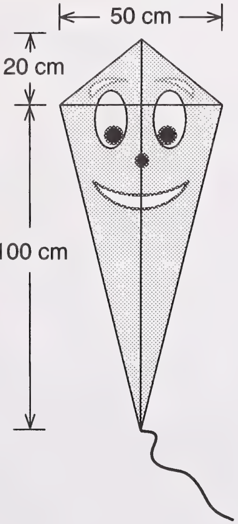
c.



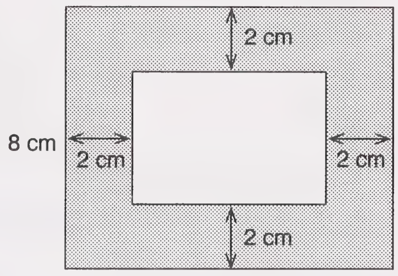
d.



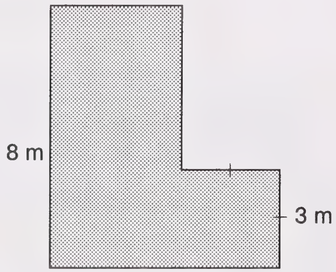
2. Calculate the area of the kite.



3. Find the area of this picture frame.



4. Mrs. Ben Zui is carpeting her living room and dining room. The cost of carpeting is $\$34.24/\text{m}^2$.



a. Find the amount of carpet needed.
b. Calculate the cost of carpeting the two rooms.

Turn to the Appendix to check your answers.

THE PYTHAGOREAN THEOREM

What Lies Ahead

In this section you will demonstrate the Pythagorean Theorem. You will also apply the Pythagorean Theorem in practical situations.

Working Together

In the sixth century B.C., a mathematician and philosopher known as Pythagoras (approximately 575 B.C.-495 B.C.) and his followers identified a numerical relationship between the lengths of the sides of a right triangle. This relationship is called the Pythagorean theorem.

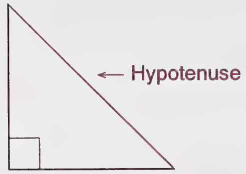
However, despite popular opinion, Pythagoras and his followers were probably not the first or only people to discover the relationship. There are indications that the principle was applied earlier by the Chinese, Egyptians, and Babylonians.

In this section, you will learn the Pythagorean theorem and its applications with right triangles.

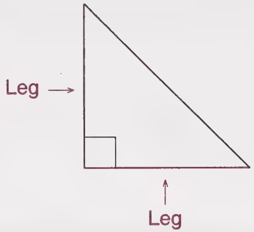
Right Triangles

Right triangles have an angle with a measure of 90°.

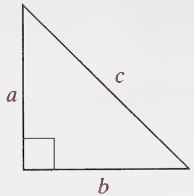
The side opposite the right angle is the **hypotenuse**. It is the longest side.



The other two sides of the triangle are the **legs**.



The sides of a right triangle are often represented to by a letter.



There is a relationship between a, b, and c, as you will discover in Practice Activity 1.

Practice Activity 1

1. Locate Pythagorean Puzzle 1 in the Appendix. Notice how the squares have been constructed on the legs and the hypotenuse of the right triangle. Square A has a side of a; therefore, the area of square A is a^2. Square B has a side of b; therefore, the area of square B is b^2. Square C has a side of c; therefore, the area of square C is c^2.

   Cut out square 1 and cover square A to verify that the squares are congruent.

   Cut out square 2 and cover square B to verify that the squares are congruent.

   Cut square 1 along the dotted lines.

   Arrange square 2 and the four pieces of square 1 to cover square C. What is the relationship between a^2, b^2, and c^2?

2. Locate Pythaogrean Puzzle 2 in the Appendix. Notice that the squares have been constructed on the legs and the hypotenuse of the triangle. Square A has a side of a; therefore, the area of square A is a^2. Square B has a side of b; therefore, the area of square B is b^2. Square C has a side of c; therefore, the area of square C is c^2.

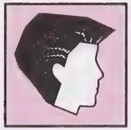
   Cut out square 3 and cover square A to verify that the squares are congruent.

   Cut out square 4 and cover square B to verify that the squares are congruent.

   Cut square 3 along the dotted lines.

   Arrange square 4 and the four pieces of square 3 to cover square C. What is the relationship between $a^2 + b^2$ and c^2?

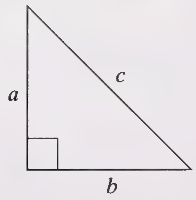
Turn to the Appendix to check your answers.



Working Together

In Practice Activity 1 you learned that there is a relationship between the sides of a right triangle.

The Pythagorean theorem states that in a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse.



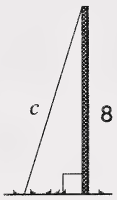
$$a^2 + b^2 = c^2$$

The Pythagorean theorem is used to solve many problems involvng triangles.

Example 1

A power pole that is 8 m high is held up by a guy wire anchored 2.5 m from its base. How long is the wire?

$$a^2 + b^2 = c^2$$
$$(2.5)^2 + 8^2 = c^2$$
$$6.25 + 64 = c^2$$
$$70.25 = c^2$$
$$c = \sqrt{70.25}$$
$$c \doteq 8.38$$

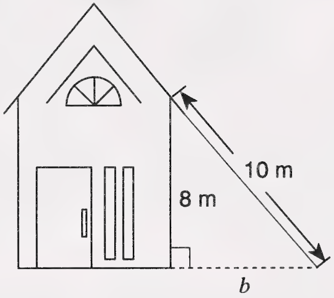


The wire is approximately 8.4 m long.

Example 2

A ladder that is 10 m long reaches the roof of a house. If it is 8 m to the roof, how far is the foot of the ladder from the house?

$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$
$$\cancel{64} - \cancel{64} \qquad -64$$
$$b^2 = 36$$
$$b = 6$$



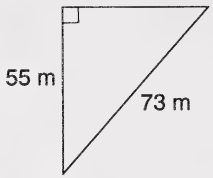
The foot of the ladder is 6 m from the house.

Practice Activity 2

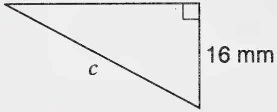
1. For each of the following triangles, calculate the length of the missing side.

a.

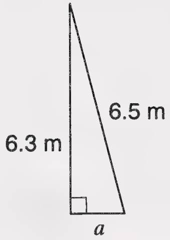
b.



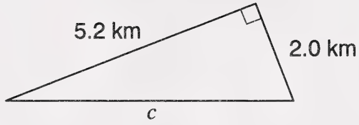
c.



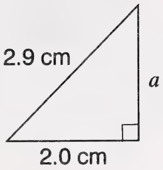
d.



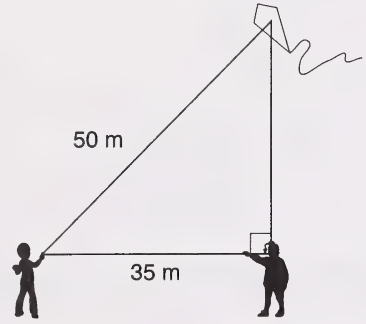
e.



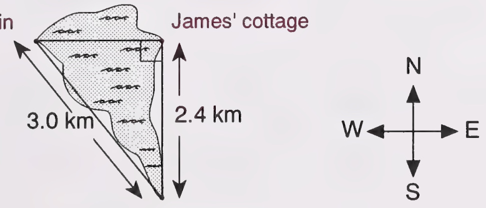
f.



2. Zoe has let out 50 m of kite string when she notices that her kite is directly above Zorna. If Zoe is 35 m from Zorna, how high is the kite above Zorna?



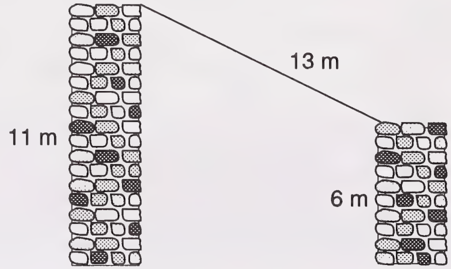
3. John's cottage is 2.4 km south of James' cottage. Jennifer's cabin is 3.0 km north-west of John's cottage. Jennifer's cabin is west of James' cottage. How far is it across the lake from Jennifer's cabin to James' cottage?



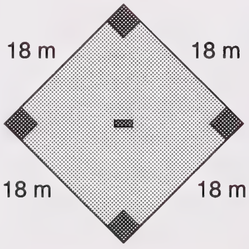
4. A box is 120 cm long and 25 cm wide. What is the length of the longest ski pole that could be packed to lie flat in the box?

5. A television screen measures 30 cm wide and 22 cm high. What is the diagonal measure of the screen?

6. Two walls are conncected by a steel cable that is 13 m long. If one wall is 11 m high and the other is 6 m high, find the distance between the walls.

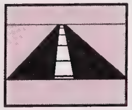


7. A player at second base must throw a ball to home plate to put a player out. How far must the player throw the ball?



Turn to the Appendix to check your answers.

RIGHT PRISMS AND CYLINDERS



What Lies Ahead

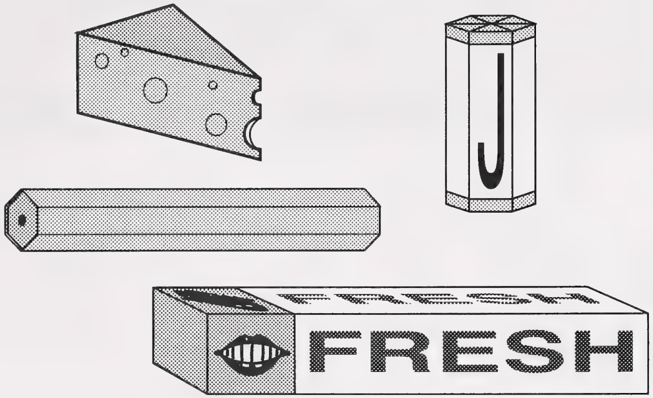
In this section you will learn these skills.

- making right prisms and cylinders from nets
- classifying right prisms and cylinders

Working Together

Right Prisms

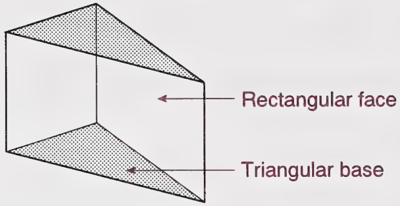
The world is full of three-dimensional objects called right prisms.



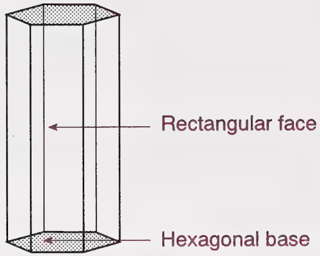
A **right prism** has two congruent **bases**. The remaining flat surfaces are **lateral faces**. The lateral faces are rectangles.

A right prism is named according to the shape of its base.

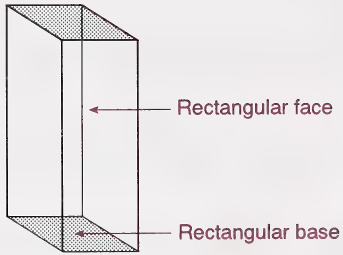
Examples



This is a triangular prism.

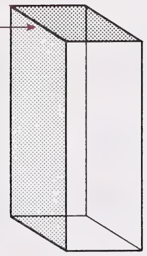


This is a hexagonal prism.

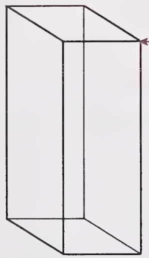


This is a rectangular prism.

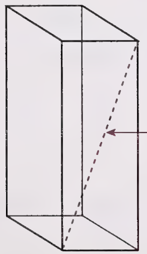
Where two faces meet is called an **edge**.



Where two edges meet is called a **vertex**.



A line segment that joins two non-adjacent vertices is called a **diagonal**.



Nets

Sometimes three-dimensional objects can be made into two-dimensional objects.

Grocery stores and moving companies often store their boxes flat and reassemble them when they are required.

A **net** is a two-dimensional figure which when folded will make a three-dimensional object. You will start this section by making right prisms and cylinders from nets.

Practice Activity 1

1. Remove the nets for a rectangular prism, triangular prism, pentagonal prism, and hexagonal prism from the Appendix.

   Glue each net on a heavy piece of paper or a light piece of cardboard. Then assemble each net by following these steps.

   Step 1: Carefully cut along the solid lines.

   Step 2: Using a ruler and a ballpoint pen, press firmly to indent the dotted lines. This process is known as "scoring" and will help you neatly fold the net along the dotted lines.

   Step 3: Fold along each dotted line to make a crease. Be sure the dotted line is on the **inside** of the crease.

   Step 4: Tape or glue the tabs so that the object is held together.

   Why is each prism given its name?

2. Examine each prism that you constructed in Question 1 and then copy and complete this table in your notebook.

Prism	Number of Bases	Number of Lateral Faces	Number of Edges	Number of Vertices
Rectangular Prism				
Triangular Prism				
Pentagonal Prism				
Hexagonal Prism				

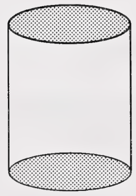
3. Which prisms have all the lateral faces congruent?

Turn to the Appendix to check your answers.

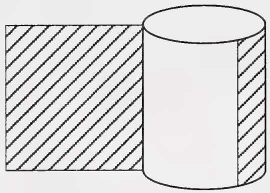
Working Together

Cylinders

This three-dimensional figure is a cylinder. A cylinder has two congruent circles for its bases.



The lateral face is not made up of a number of rectangles. Instead, the lateral face is one large rectangle. This can be seen by peeling the label off an empty food can.



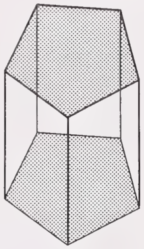
Practice Activity 3

1. Remove the net for a cylinder from the Appendix. Glue it to heavy paper or light cardboard and assemble it.

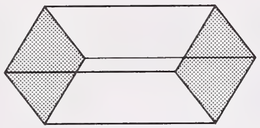
   a. How is the length of the rectangle related to the circular base?
   b. How is the width of the rectangle related to the cylinder?

2. Name each of the following objects.

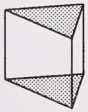
a.



b.



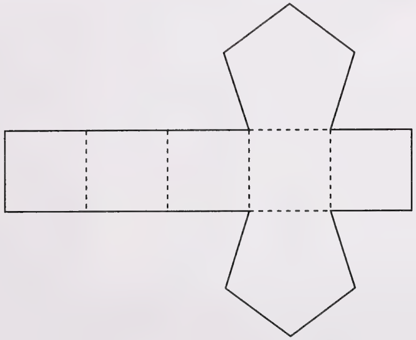
c.



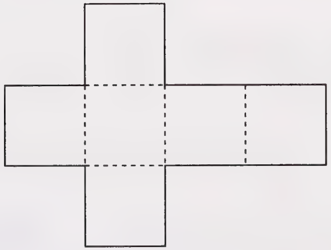
d.

3. Identify each of the following objects from the nets shown.

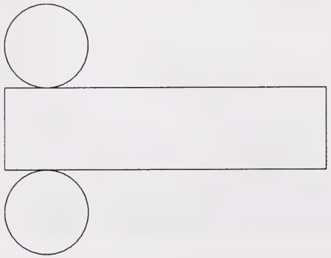
a.



b.

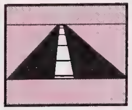


c.



Turn to the Appendix to check your answers.

SURFACE AREA



What Lies Ahead

In this section you will learn to find the surface area of a right prism or cylinder.

Working Together

To begin this section there is a video activity.

Video Activity

View the video *MATHWAYS: Areas*. It will review the area of a polygon and discuss surface area.

Nets and Surface Area

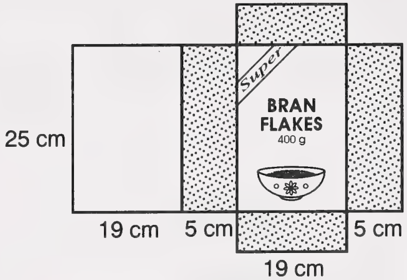
In the previous section you studied nets to help construct and classify right prisms. This study of nets will help you understand surface area.

Example 1

Find the surface area of this cereal box.

Solution

The net of the box looks like this.



The net is made up of six rectangles. There are two rectangles that are 19 cm by 5 cm. There are two rectangles that are 19 cm by 25 cm. There are two rectangles that are 5 cm by 25 cm.

To find the surface area of the cereal box, find the area of each rectangle.

Calculate the area of one rectangle that is 19 cm by 5 cm.

$$\begin{aligned} A &= b \times h \\ &= 19 \times 5 \\ &= 95 \end{aligned}$$

Calculate the area of one rectangle that is 19 cm by 25 cm.

$$\begin{aligned} A &= b \times h \\ &= 19 \times 25 \\ &= 475 \end{aligned}$$

Calculate the area of one rectangle that is 5 cm by 25 cm.

$$\begin{aligned} A &= b \times h \\ &= 5 \times 25 \\ &= 125 \end{aligned}$$

Calculate the surface area of the cereal box.

$$\begin{aligned} SA &= (2 \times 95) + (2 \times 475) + (2 \times 125) \\ &= 190 + 950 + 250 \\ &= 1390 \end{aligned}$$

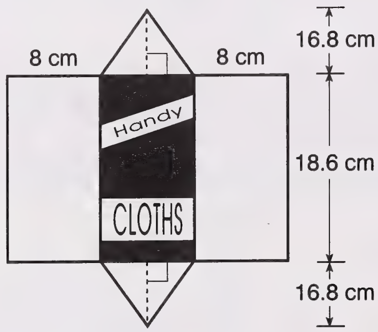
The surface area of the cereal box is 1390 cm^2.

Example 2

Find the surface area of the box of cloths.

Solution

The net of the box looks like this.



There are two congruent triangles. There are three congruent rectangles.

To find the surface area of the box, find the area of each face.

Calculate the area of one triangle.

$$
\begin{aligned}
A &= \frac{b \times h}{2} \\
&= \frac{8 \times 16.8}{2} \\
&= \frac{134.4}{2} \\
&= 67.2
\end{aligned}
$$

Calculate the area of one rectangle.

$$
\begin{aligned}
A &= b \times h \\
&= 8 \times 18.6 \\
&= 148.8
\end{aligned}
$$

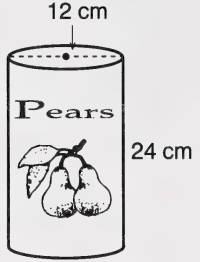
Calculate the surface area of the box.

$$
\begin{aligned}
SA &= (2 \times 67.2) + (3 \times 148.8) \\
&= 134.4 + 446.4 \\
&= 580.8
\end{aligned}
$$

The surface area of the box is 580.8 cm^2.

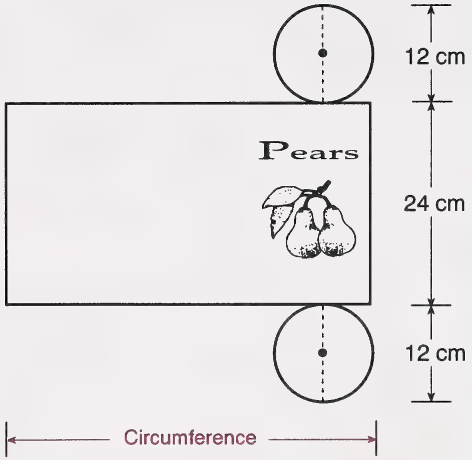
Example 3

Find the surface area of the can.



Solution

The net of the can looks like this . There is one rectangle and two congruent circles.



To find the surface area of the can, find the area of each base and the rectangle.

Find the area of one circle.

$$A = \pi r^2$$
$$\doteq 3.14 \times 6^2$$
$$\doteq 3.14 \times 36$$
$$\doteq 113.04$$

Find the area of the rectangle.

$$C = \pi d$$
$$\doteq 3.14 \times 12$$
$$\doteq 37.68$$

$$A = b \times h$$
$$\doteq 37.68 \times 24$$
$$\doteq 904.32$$

Calculate the area of the can.

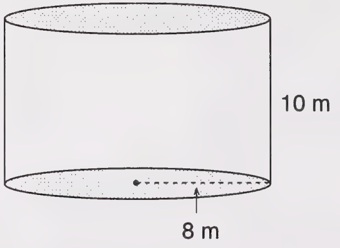
$$SA \doteq (2 \times 113.04) + 904.32$$
$$\doteq 1130.4$$

The surface area of the can is about 1130.4 cm^2.

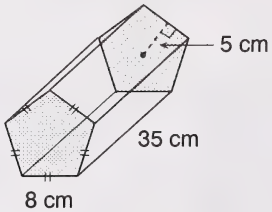
Practice Activity 1

1. Calculate the surface area for each of the following prisms.

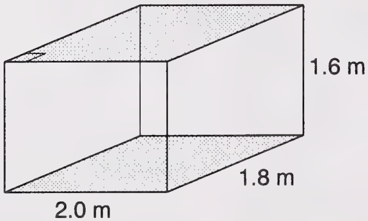
a.



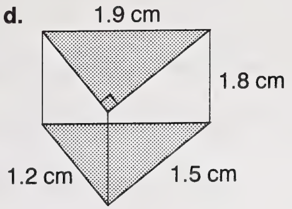
b.



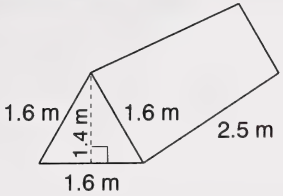
c.



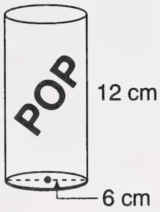
d.



2. A tent has the dimensions shown. If the tent also has a canvas floor, how much canvas was used in its construction? Assume there was no wastage.



3. A pop can has a diameter of 6 cm and a height of 12 cm. What was the area of the metal sheet used in its construction? Assume there was no wastage.



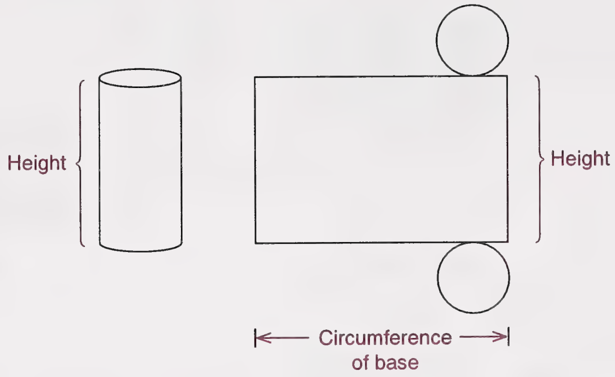
Turn to the Appendix to check your answers.

Working Together

The Formula for the Surface Area of a Cylinder

A formula can be developed for the surface area of a cylinder.

Look at this net of a cylinder. The net is made up of two bases which are congruent circles and one rectangle. The width of the rectangle is the height of the cylinder. The length of the rectangle is equal to the circumference of the base.



This relationship can be expressed by a formula.

Surface area of cylinder = 2 × area of base of cylinder + circumference of base of cylinder × height of cylinder

or $SA = 2B + C \times H$

Example

What is the surface area of this can?

Solution

$B = \pi r^2$
$\doteq 3.14 \times 4^2$
$\doteq 3.14 \times 16$
$\doteq 50.24$

$d = 8$
So, $r = 4$

$C = \pi d$
$\doteq 3.14 \times 8$
$\doteq 25.12$

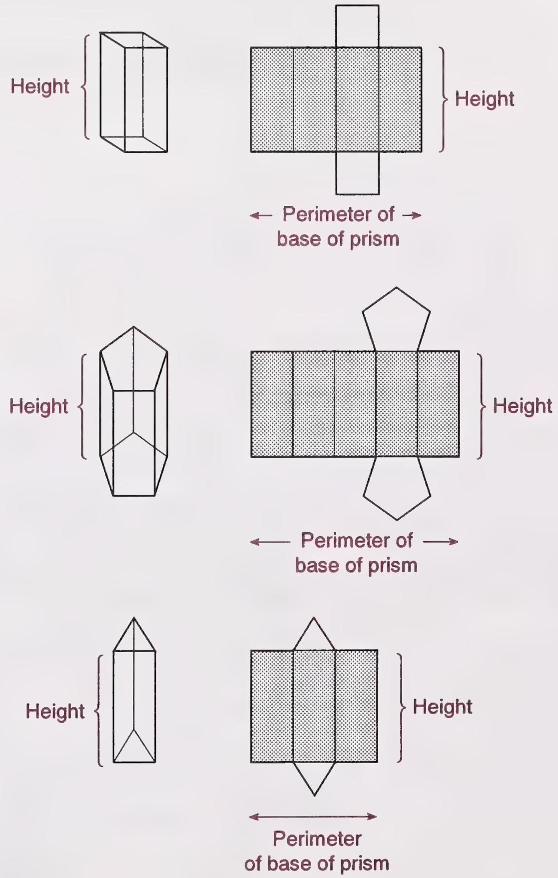
$SA = 2B + C \times H$
$\doteq (2 \times 50.24) + (25.12 \times 10)$
$\doteq 100.48 + 251.2$
$\doteq 351.68$

The surface area of the can is about 351.68 cm^2.

Formula for the Surface Area of a Prism

A formula can be developed for the surface area of any prism.

Look at the nets of these prisms. Each net is made up of the two bases which are congruent and four rectangles. The four rectangles form one large rectangle. The width of the large rectangle is equal to the height of the prism. The length of the large rectangle is equal to the perimeter of the base.



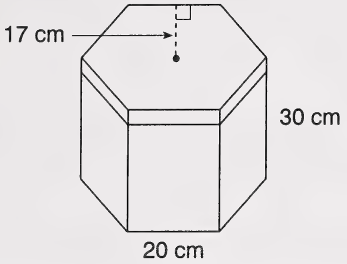
The relationship can be expressed by a formula.

Surface area of prism = 2 × area of base of prism + perimeter of base of prism × height of prism

or $SA = 2B + P \times H$

Example

Find the surface area of this hat box.



Solution

Calculate the area of the base of the prism.

$$\begin{aligned} B &= \frac{n \times s \times a}{2} \\ &= \frac{6 \times 20 \times 17}{2} \\ &= \frac{2040}{2} \\ &= 1020 \end{aligned}$$

Calculate the perimeter of the base of the prism.

$$\begin{aligned} P &= ns \\ &= 6 \times 20 \\ &= 120 \end{aligned}$$

Calculate the surface area.

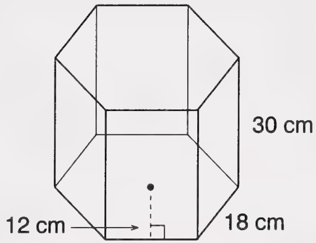
$$\begin{aligned} SA &= 2B + P \times H \\ &= (2 \times 1020) + (120 \times 30) \\ &= 2040 + 3600 \\ &= 5640 \end{aligned}$$

The surface area of the hat box is 5640 cm^2.

Practice Activity 2

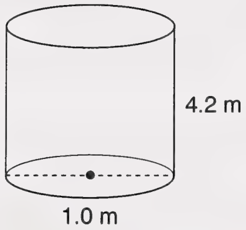
1. Calculate the surface area for each of the following.

a.

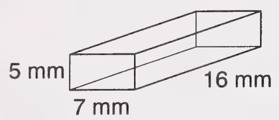


(base is a regular polygon)

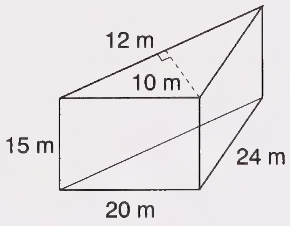
b.



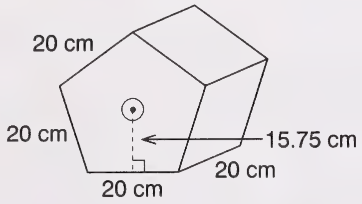
c.



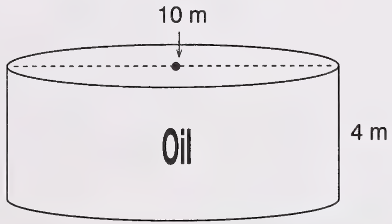
d.



2. Calculate the surface area of this bird house. The bases are regular pentagons.



3. Calculate the surface area of the oil tank.



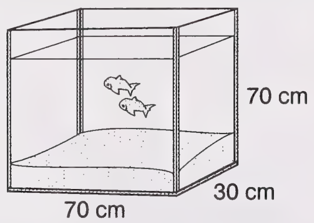
Turn to the Appendix to check your answers.

Working Together

Some problems require you to calculate the surface area of prisms and cylinders that are open.

Example

What is the surface area of the exterior of this aquarium?



Solution

The aquarium has only one base.

$$\begin{aligned} P &= 2\ell + 2w \\ &= (2\times 70) + (2\times 30) \\ &= 140 + 60 \\ &= 200 \end{aligned}$$

$$\begin{aligned} SA &= B + P \times H \\ &= (70\times 30) + (200 \times 70) \\ &= 2100 + 14\,000 \\ &= 16\,100 \end{aligned}$$

The surface area of the exterior of the aquarium is 16 100 cm^2.

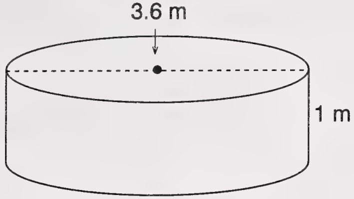
Note: $1\ m^2 = 10\,000\ cm^2$

So, $16\,100\ cm^2 = 1.61\ m^2$

The surface area of the exterior of the aquarium is 1.61 m^2.

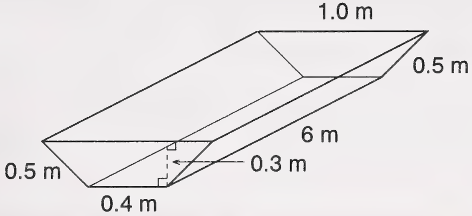
Practice Activity 3

1. This cylindrical pool has vinyl material on the inside. How much vinyl material was used? Assume there was no wasted material.



2. A room is 3.7 m by 4.4 m, and has a height of 2.5 m. If only the walls are to be painted, what area needs painting?

3. A water trough with the dimensions shown is to be constructed from galvanized steel. If the trough has no top, what area of steel is required for its construction? Assume there is no wastage.



Turn to the Appendix to check your answers.

Did You Know?

Several mathematicians have made some most unusual things. Ardhimedes, who lived around 250 B.C. and who was a great student of mathematics, made some huge hooks to lift enemy ships out of the water. He also made some long poles that could be used to throw rocks at enemy ships.

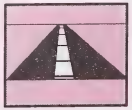
Heron lived several hundred years after Archimedes. When he wasn't studying triangles and circles, he was making things. He made a toy fire engine and an organ. He fixed the temple doors so they would open as supermarket doors open today. People loved to see themselves in his mirrors that would show them upside down.

Alhazen was an Arab mathematician who lived around 1000 A.D.. He was very proud of his knowledge of geometry and once boasted that he could make a machine to control the floods from the Nile River. The ruler in Egypt was interested in this machine and sent for Alhazen. The Arabian knew his idea wouldn't work, but he was afraid to tell the ruler. He pretended to have lost his mind so that he wouldn't be expected to make the machine. He learned his lesson about boasting because he had to go on pretending until the ruler died.

A short time before Columbus discovered America, a mathematician lived near a place in Germany called King's Mountain. This man is known as Regiomontanus, which means "king's mountain" in Latin. He worked on the calendar, studied geometry, and wrote about astronomy. Most of the people of his day did not know about his mathematics, but they did enjoy seeing his mechanical eagle flying around the mountain with its big wings flapping.

Pascal, a French mathematician of the seventeenth century, liked to study mathematics and religion. At one time he put his mind to the practical matter of making an adding machine. His machine could add six-digit numbers. Some of his machines are preserved in a museum in Paris.

VOLUME



What Lies Ahead

In this section you will develop and use formulas to calculate the volume of right prisms and cylinders.

Working Together

To begin this section there is a video activity.

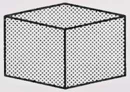
Video Activity

Watch the video *MATHWAYS: Volumes*. It provides a good introduction to calculating the volume of prisms and cylinders.

Working Together

As you learned from the video, volume is a measurement of how much space an object occupies.

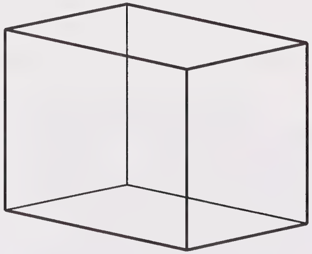
You can find volume directly by counting the number of cubic units.



1 cubic unit

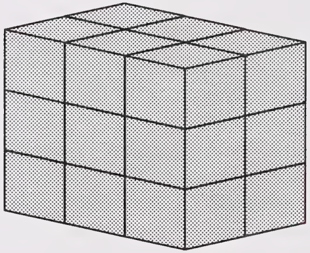
Example

What is the volume of this rectangular prism?



Solution

This rectangular prism can be filled with cubic units. To find the volume, count the cubic units.

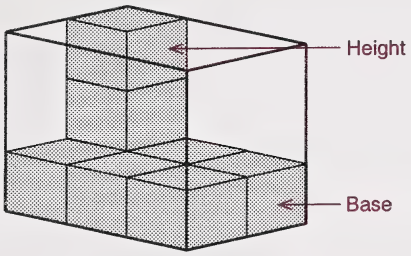


Some of the cubic units are hidden in this diagram.

This rectangular prism has a volume of 18 cubic units.

You can find the volume without counting each cubic unit.

Simply count the blocks in the first layer (the area of the base) and multiply by the number of layers (the height).



There are six blocks in the first layer (the area of the base) and there are three layers (the height).

So, the volume of the rectanglular prism is 18 cubic units.

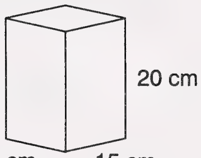
You can express this rule with a formula.

$$V = B \times H$$

Note: Upper case letters are used to represent the area of the base and the height of the prism.

Example

$$\begin{aligned} V &= B \times H \\ &= (10 \times 15) \times 20 \\ &= 3000 \end{aligned}$$



The volume is 3000 cm^3.

Using the Formula

You can use the formula to find the volume of other rectangular prisms indirectly.

Example

What is the volume of this chocolate box?

Solution

$$\begin{aligned} B &= \frac{b \times h}{2} \\ &= \frac{3.6 \times 3.1}{2} \\ &= 5.58 \end{aligned}$$

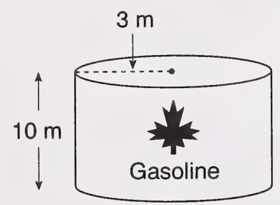
$$\begin{aligned} V &= B \times H \\ &= 5.58 \times 16.3 \\ &= 91.0 \end{aligned}$$

The volume of the box is 91 cm^3.

You can use the formula $V = B \times H$ to find the volume of cylinders indirectly.

Example 2

What is the volume of this storage tank?



Solution

$B = \pi r^2$
$\doteq 3.14 \times 3$
$\doteq 9.42$

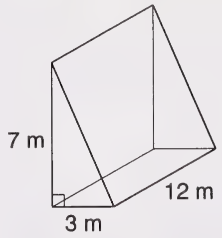
$V = B \times H$
$\doteq 9.42 \times 10$
$\doteq 94.2$

The volume of the storage tank is about 94.2 m^3.

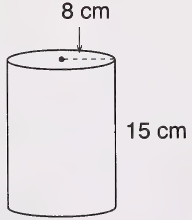
Practice Activity 1

1. Calculate the volume of each of the following.

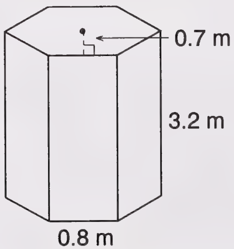
a.



b.



c.



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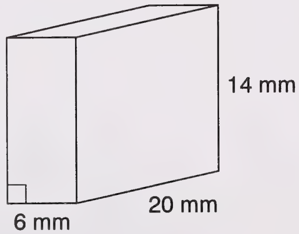
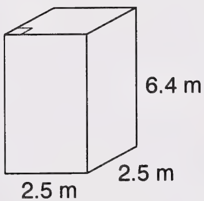
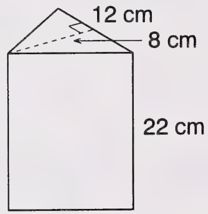


e.



f.



2. How much wax is needed to make a cylindrical candle with a radius of 3 cm and a height of 10 cm?

3. The box on a dump truck is 3.2 m long, 2.6 m wide, and 1.8 m high. What is the maximum volume of soil it could carry without having any above the top of the box?

4. A plastic garbage can is 0.55 m in diameter and 1.0 m high. What is its volume?

5. A patio is 6.9 m long and 5.8 m wide. Kavita wishes to cover this with concrete which is 20 cm thick. How much will it cost her if concrete is $15.00 per cubic metre?

Turn to the Appendix to check your answers.

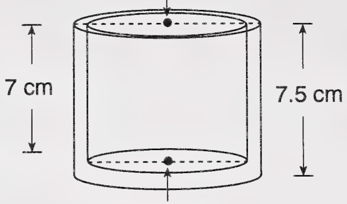
Working Together

Consumer Packaging

Some containers have quite a large difference between their outside volumes and their inside volumes.

Example

Beauty Hands Inc. sells their facial cream in a jar. The outside of the jar has a diameter of 10 cm and a height of 7.5 cm. The inside of the jar has a diameter of 9.5 cm and a height of 7 cm.



Compare the inside volume and the outside volume.

Outside Volume

$$B = \pi r^2 \doteq 3.14 \times 5^2 \doteq 3.14 \times 25 \doteq 77.5$$

$$V = B \times H \doteq 77.5 \times 7.5 \doteq 581.25$$

The outside volume of the jar is about 581.25 cm^3.

Inside Volume

$$B = \pi r^2 \doteq 3.14 \times (4.75)^2 \doteq 3.14 \times 22.56 \doteq 70.84$$

$$V = B \times H \doteq 70.84 \times 7 \doteq 495.88$$

The inside volume is about 495.88 cm^3.

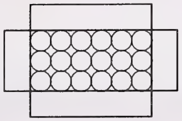
Difference

$581.25 - 495.88 = 85.37$

The difference between the outside volume and the inside volume is $85.37\ cm^3$.

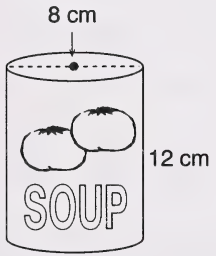
Practice Activity 2

1. Eighteen cans of soup are packaged in a rectangular box as shown.



Top view of carton

Each can is 12 cm high and has a diameter of 8 cm.



The cardboard carton is 24 cm wide, 48 cm long, and 12 cm high.

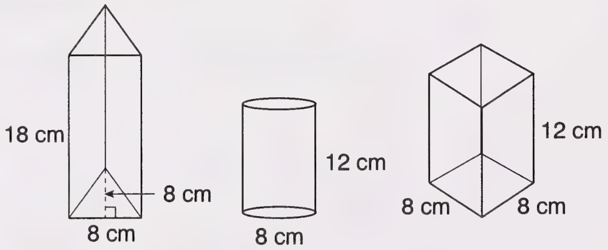
How much empty space is in the carton?

2. a. Which of the following containers do you think is most attractive?

   b. Which one is easier to handle?

   c. Which one looks larger?

   d. Which one is larger?



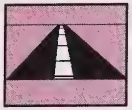
3. A candy bar company has been selling a chocolate bar that is 5 cm by 9 cm by 1 cm.

   a. If the manufacturers reduce the thickness of the candy bars from 1 cm to 0.9 cm, is the customer likely to notice the change?

   b. By what percent will the volume of the chocolate bar be reduced?

Turn to the Appendix to check your answers.

MODULE CONCLUSION



What Lies Ahead

In the module conclusion you will review the module and do the module assignment.

Working Together

In this Module you reviewed skills previously developed in Mathematics 7 and 8 and learned these skills.

- classifying pairs of angles as supplementary angles, complementary angles, adjacent angles, and opposite angles
- determining the sum of the angles of a triangle and the sum of the angles in any polygon
- making regular polygons
- copying segments, angles, and triangles
- constructing triangles by copying segments
- bisecting segments and angles
- constructing angles of 90°, 45°, 60°, and 30°
- constructing perpendicular lines
- developing and using formulas to calculate the perimeter of a polygon and the circumference of a circle indirectly
- developing and using formulas to calculate area indirectly
- demonstrating and applying the Pythagorean theorem
- making right prisms and cylinders from nets
- classifying right prisms and cylinders
- finding the surface area of a right prism or cylinder
- developing and using formulas to calculate the volume of right prisms and cylinders

Now that you have studied Module 6, you should be ready to do the assignment for Module 6.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX

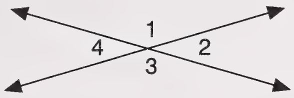
- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Acute angle: an angle measuring less than 90°

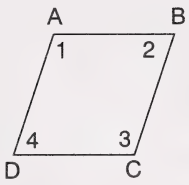
Acute triangle: a triangle with three acute angles

Adjacent angles of intersecting lines: two angles that share a common vertex and common ray



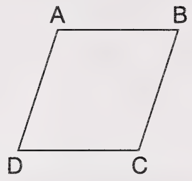
$\angle 1$ and $\angle 2$ are adjacent angles.
$\angle 2$ and $\angle 3$ are adjacent angles.
$\angle 3$ and $\angle 4$ are adjacent angles.
$\angle 4$ and $\angle 1$ are adjacent angles.

Adjacent angles of polygons: two angles whose vertices are adjacent



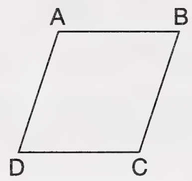
$\angle 1$ and $\angle 2$ are adjacent angles.
$\angle 2$ and $\angle 3$ are adjacent angles.
$\angle 3$ and $\angle 4$ are adjacent angles.
$\angle 4$ and $\angle 1$ are adjacent angles.

Adjacent sides of a polygon: two sides which have a common endpoint (vertex)



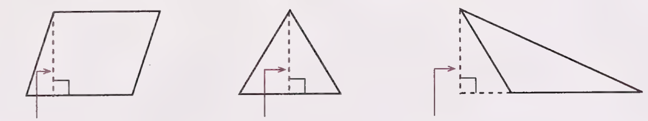
$\overline{AB}$ and $\overline{BC}$ are adjacent sides.
$\overline{BC}$ and $\overline{CD}$ are adjacent sides.
$\overline{CD}$ and $\overline{DA}$ are adjacent sides.
$\overline{DA}$ and $\overline{AB}$ are adjacent sides.

Adjacent vertices of a polygon: two vertices which are endpoints of the same side

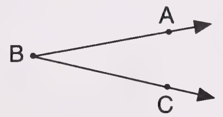


A and B are adjacent vertices.
B and C are adjacent vertices.
C and D are adjacent vertices.
D and A are adjacent vertices.

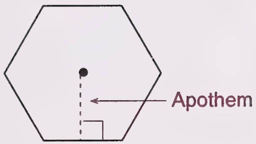
Altitude: a line segment from a vertex of a polygon perpendicular to the opposite side or extension of that side



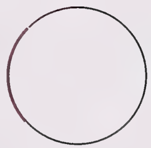
Angle: a set of points consisting of two rays extending from a common endpoint or vertex



Apothem of a regular polygon: the line segment from the centre of the regular polygon perpendicular to one of the sides

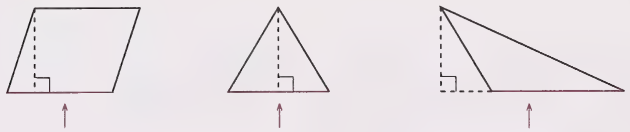


Arc: a portion of a circle

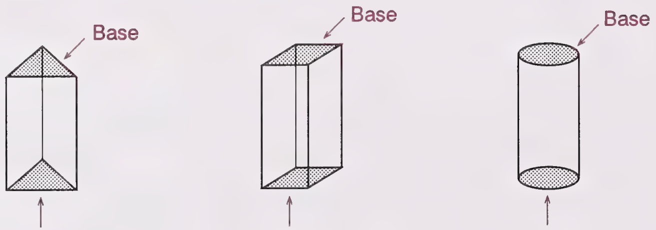


Area: a measurement in square units of the amount of surface of a figure

Base of a polygon: the sides of the polygon from which the height of the polygon is measured

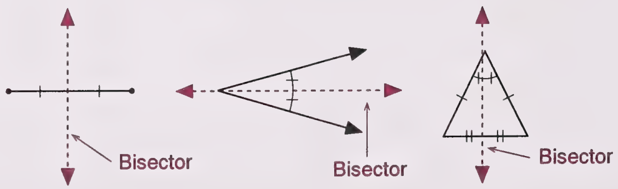


Base of a right prism or cylinder: the face of the right prism or cylinder from which the height is measured

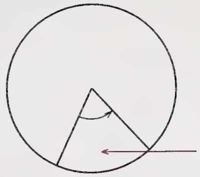


A prism and cylinder each have two bases.

Bisector: the line which divides a segment, angle, or figure in half



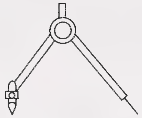
Central angle of a circle: an angle formed by two radii of a circle



Circumference: the distance around a circle

Closed curve: a connected set of points with no endpoints

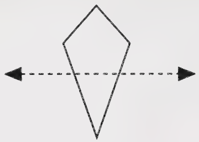
Compass (pair of compasses): a mathematical tool



Complementary angles: two angles with a combined measure equal to 90°

Concave polygon: a polygon in which a straight line can cut through more than two sides

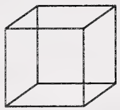
Convex polygon: a polygon in which a straight line can cut through only two sides



Congruent: having the same size and shape

Construction: a figure drawn using only two tools: a straightedge and a compass

Cube: a rectangular prism with all six faces congruent

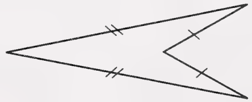


Curve: a connected set of points

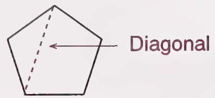
Decagon: a polygon with ten sides and ten angles

Degree of an angle: a unit for measuring angles

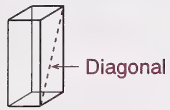
Deltoid: a concave quadrilateral with two pairs of congruent adjacent sides



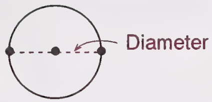
Diagonal of a polygon: a line segment which connects two nonadjacent vertices



Diagonal of a prism: a line segment which connects two nonadjacent vertices

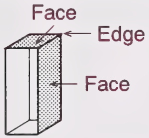


Diameter: a line segment which joins two points on a circle and passes through the centre of a circle



Dodecagon: a polygon with 12 sides and 12 angles

Edge of a prism: the line segment where two faces of a prism meet



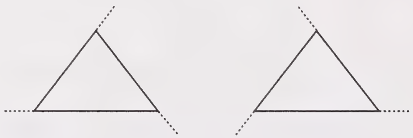
A rectangular prism has twelve edges.

Endpoint: a point at the end of a line segment or ray

Equiangular triangle: a triangle with three congruent angles

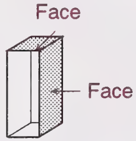
Equilateral triangle: a triangle with three congruent sides

Exterior angle of a polygon: an angle at a vertex of a polygon whose rays fall along the side of the polygon and the extension of the adjacent side



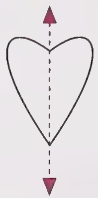
A triangle has six exterior angles – two at each vertex.

Face: a flat surface on a three-dimensional object



A rectangular prism has six faces.

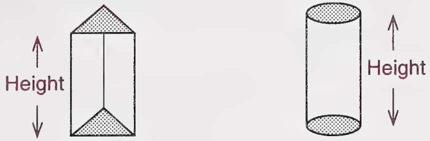
Flip symmetry: the property of being symmetrical, able to be divided into two congruent parts that are flip images of each other



A heart has flip symmetry.

Height of a polygon: the perpendicular distance from a vertex of the polygon to the opposite base

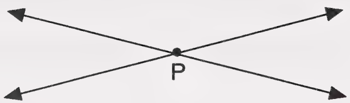
Height of a right prism or cylinder: the perpendicular distance between the bases



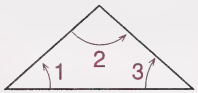
Heptagon: a polygon with seven sides and seven angles

Hexagon: a polygon with six sides and six angles

Intersecting lines: two lines that meet and share a common point



Interior angle of a polygon: an angle at a vertex of a polygon whose rays fall along the sides of the polygon

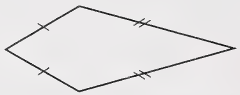


A triangle has three interior angles – one at each vertex.

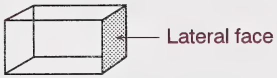
Isosceles triangle: a triangle with two congruent sides

Isosceles trapezoid: a trapezoid with two congruent sides

Kite: a convex quadrilateral with two pairs of congruent adjacent sides



Lateral faces of right prisms: the faces that are not bases of a right prism

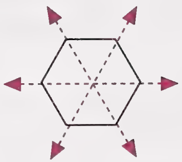


A rectangular prism has four lateral faces.

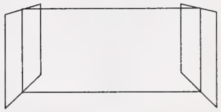
Line: a set of points which extends in a straight path infinitely in both directions

Line segment: a part of a line with two endpoints

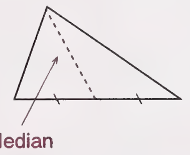
Line of symmetry: a line that divides a geometric figure into congruent parts that are flip images of each other



Mira: a mathematical tool made of plexiglass, a transparent plastic mirror



Median: a line segment drawn from a vertex of a polygon to the midpoint of the opposite side



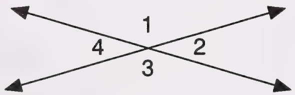
Midpoint: the point which divides a line segment into two congruent parts

Nonagon: a polygon with nine sides and nine angles

Obtuse angle: an angle measuring between 90° and 180°

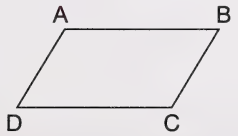
Obtuse triangle: a triangle with one obtuse angle

Opposite angles of intersecting lines: the pair of nonadjacent angles formed by two intersecting lines



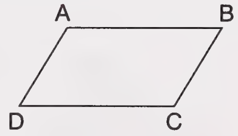
$\angle 1$ and $\angle 3$ are opposite angles.
$\angle 2$ and $\angle 4$ are opposite angles.

Opposite angles of a quadrilateral: the nonadjacent angles of a quadrilateral



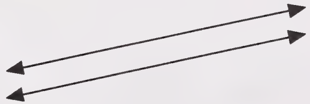
$\angle A$ and $\angle C$ are opposite angles.
$\angle B$ and $\angle D$ are opposite angles.

Opposite sides of a quadrilateral: the nonadjacent sides of a quadrilateral



$\overline{AB}$ and $\overline{CD}$ are opposite sides.
$\overline{AD}$ and $\overline{BC}$ are opposite sides.

Parallel lines: two lines in a plane that do not intersect

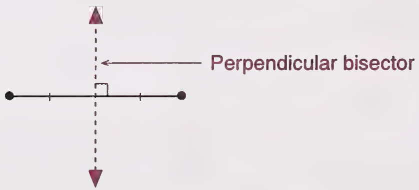


Parallelogram: a quadrilateral with opposite sides parallel

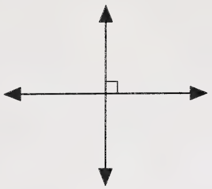
Pentagon: a polygon with five sides and five angles

Perimeter: the distance around a closed curve

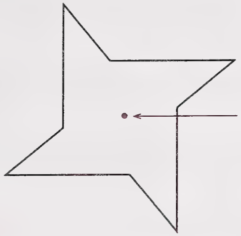
Perpendicular bisector: a line that bisects a segment at right angles



Perpendicular lines: lines that intersect at right angles

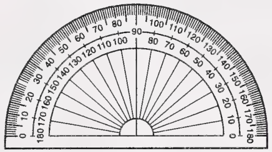


Point of symmetry: a point about which a figure can be turned so it fits onto itself more than once in a full turn



Polygon: a simple closed curve made of three or more line segments

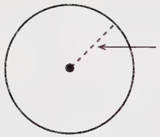
Protractor: a mathematical tool for measuring angles



Octagon: a polygon with eight sides and eight angles

Quadrilateral: a polygon with four sides and four angles

Radius: a line segment that joins the centre of a circle with any point on the circumference



Ray: a set of points that extends in a straight line infinitely in one direction

Rectangle: a parallelogram with four 90° angles

Reflex angle: an angle between 180° and 360°

Regular polygon: a polygon with four congruent and all angles congruent

Rhombus: a parallelogram with four congruent sides

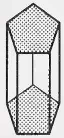
Right angle: an angle measuring 90°

Right prism: a three-dimensional solid with two congruent bases and rectangular lateral faces

Prisms are named according to the shape of their bases.

Rectangular prism

Triangular prism



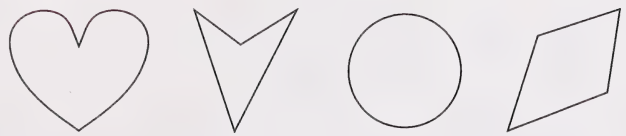
Pentagonal prism

Right triangle: a triangle with a 90° angle

Scalene triangle: a triangle with no congruent sides

Similar: having the same shape but a different size

Simple closed curve: a closed curve with no crossovers



Square: a parallelogram with four 90° angles and four congruent sides

Straight angle: an angle measuring 180°

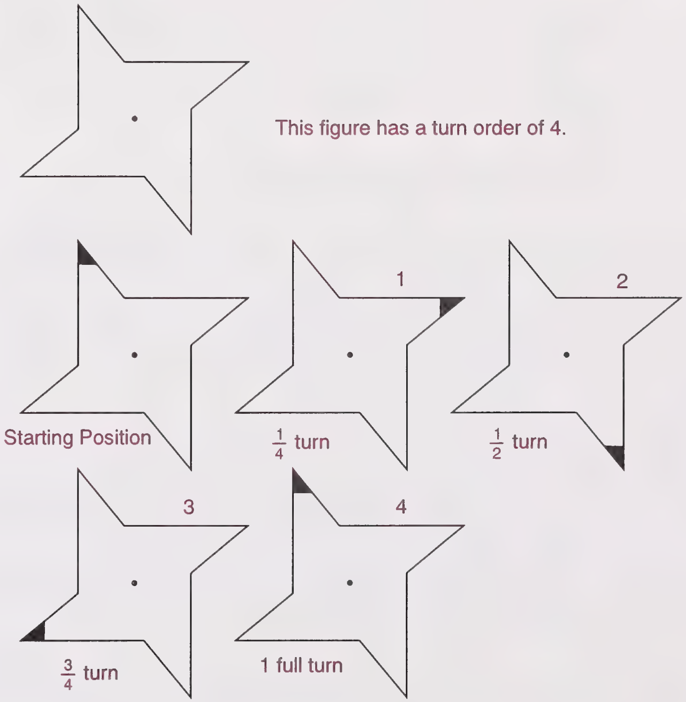
Straightedge: a mathematical tool without any scale used for making line segments

Supplementary angles: two angles with a combined measure of 180°

Triangle: a polygon with three sides and three angles

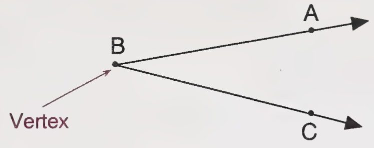
Trapezoid: a quadrilateral with one pair of parallel sides

Turn order: the number of times a figure can fit onto itself in a full turn

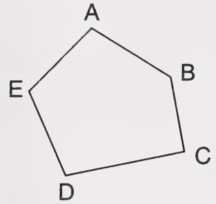


Turn Symmetry: the property of being symmetrical, able to be turned so that the figure fits onto itself more than once in a full turn

Vertex of an angle: the common point of the two rays of an angle

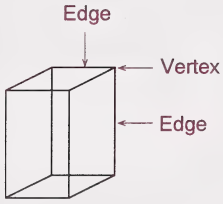


Vertex of a polygon: the common endpoint of two sides in a polygon



A, B, C, D and E are vertices.

Vertex of a prism: the common endpoint of two edges of a prism



A right rectangular prism has eight vertices.

Volume: a measure in cubic units of the amount of space in a three-dimensional object

Suggested Answers

Section 1: Review

1. **a.** iii **b.** iv **c.** i **d.** v **e.** ii **f.** vii **g.** vi

2. **a.** false **b.** false **c.** true **d.** false **e.** true
   f. true **g.** true **h.** false **i.** false **j.** false
   k. false **l.** false **m.** true **n.** true **o.** false
   p. false **q.** true

3. C, E: Polygons are simple closed curves made of three or more line segments.

4. **a.** pentagon **b.** nonagon **c.** hexagon
   d. decagon **e.** triangle **f.** quadrilateral
   g. heptagon **h.** octagon

5. **a.** obtuse triangle **b.** right triangle **c.** acute triangle

6. **a.** scalene triangle
   b. equilateral triangle or regular triangle
   c. isosceles triangle
   d. equiangular triangle or regular triangle

7.

Kind of Triangle	Number of Lines of Symmetry	Order of Turn Symmetry
Equilateral	3	3
Isosceles	1	N/A
Scalene	0	N/A

8. a. trapezoid b. square c. parallelogram
d. rhombus e. rectangle f. quadrilateral
g. kite h. deltoid i. isosceles trapezoid

9.

Kind of Quadrilateral	Number of Lines of Symmetry	Order of Turn Symmetry
Quadrilateral	N/A	N/A
Kite	1	N/A
Deltoid	1	N/A
Trapezoid	N/A	N/A
Isosceles Trapezoid	1	N/A
Parallelogram	N/A	2
Rhombus	2	2
Rectangle	2	2
Square	4	4

10.

Kind of Quadrilateral	Are Diagonals Congruent?	Do Diagonals Bisect Each Other?	Do Diagonals Bisect Each Other at 90° Angles?
Quadrilateral	No	No	No
Kite	No	No	Yes
Trapezoid	No	No	No
Isosceles Trapezoid	Yes	No	No
Parallelogram	No	Yes	No
Rhombus	No	Yes	Yes
Rectangle	Yes	Yes	No
Square	Yes	Yes	Yes

Section 1: Pretest

See your learning facilitator to check your answers.

Section 2: Practice Activity 1

1. a. $\angle BEC$ b. $\angle AEG$ or $\angle BEF$
c. $\angle DEF$ d. $\angle AEG$ or $\angle BEF$

2. a. $\angle ONS$ or $\angle RNM$ b. $\angle RPO$ or $\angle SPQ$
c. $\angle QPR$ or $\angle SPO$ d. $\angle QOU$

3. a. $x = 15°$ b. $x = 115°$ c. $x = 138°$, $y = 42°$, $z = 138°$ d. $x = 48°$, $y = 42°$, $z = 138°$

e. $x = 20°$ f. $x = 58°$, $y = 61°$ g. $x = 80°$ h. $x = 60°$, $y = 120°$

Section 2: Practice Activity 2

Print Alternative

1. a. ∠CAD or ∠CAE b. ∠DAE
   c. ∠BAC or ∠DAE d. ∠BAC

2. a. ∠2 or ∠4 b. ∠3 c. ∠5 or ∠7 d. ∠6

3. a. ∠1 or ∠3 b. ∠4 c. ∠2 or ∠4 d. ∠1

4. a. ∠1 = 90°, ∠2 = 90°, ∠3 = 90°, ∠4 = 90°
   b. ∠1 = 43°, ∠2 = 137°, ∠3 = 43°, ∠4 = 137°
   c. ∠1 = 30°, ∠2 = 150°, ∠3 = 30°, ∠4 = 150°

5. a. The opposite angles are congruent.
   b. The adjacent angles are supplementary.

6. a. ∠A = 80°, ∠B = 130°, ∠C = 50°, ∠D = 100°
   b. ∠E = 120°, ∠F = 60°, ∠G = 120°, ∠H = 60°
   c. ∠I = 90°, ∠J = 90°, ∠K = 90°, ∠L = 90°
   d. ∠M = 90°, ∠N = 90°, ∠O = 90°, ∠P = 90°
   e. ∠Q = 120°, ∠R = 60°, ∠S = 120°, ∠T = 60°
   f. ∠U = 100°, ∠V = 140°, ∠W = 100°, ∠X = 20°
   g. ∠W = 110°, ∠X = 90°, ∠Y = 120°, ∠Z = 40°

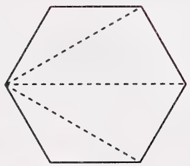
7. a. In the parallelogram, rectangle, square, and rhombus, all pairs of opposite angles are congruent.
   b. In the kite some pairs of opposite angles are congruent.
   c. In the trapezoid and the quadrilateral no pairs of opposite angles are congruent.
   d. All pairs of adjacent angles are supplementary in the parallelogram, rectangle, square, and rhombus.
   e. Some pairs of adjacent angles are supplementary in the trapezoid.
   f. No pair of adjacent angles are supplementary in the kite.

Computer Alternative

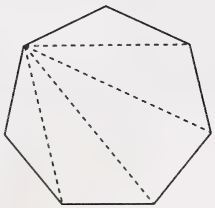
9. computer corrected

Section 3: Practice Activity 1

1. a.



b.



2. a.

Polygon	Number of Sides	Number of Diagonals Needed to Make the Polygon Rigid
Triangle	3	0
Quadrilateral	4	1
Pentagon	5	2
Hexagon	6	3

b. $d = n - 3$

c.

Polygon	Number of Sides	Number of Diagonals Needed to Make the Polygon Rigid
Heptagon	7	4
Octagon	8	5
Nonagon	9	6
Decagon	10	7

Section 3: Practice Activity 2

Print Alternative

1. The angles form a straight angle of 180°, so the sum of the angles of the triangle is 180°.

2. a. $x = 40°$ b. $x = 45°$ c. $x = 60°$ d. $x = 20°$

Computer Alternative

3. computer corrected

Section 3: Practice Activity 3

1. a. two triangles b. three triangles c. four triangles
   d. five triangles e. six triangles

2. a.

Kind of Polygon	Number of Sides	Number of Triangles	Sum of the Measures of the Angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°

b. $t = n - 2$ c. $a = (n-2) \times 180$

3. a.
$$\begin{aligned} a &= (n-2)\times 180 \\ &= (9-2)\times 180 \\ &= 7\times 180 \\ &= 1260 \end{aligned}$$
The sum of the angles in a nonagon is 1260°.

b.
$$\begin{aligned} a &= (n-2)\times 180 \\ &= (10-2)\times 180 \\ &= 8\times 180 \\ &= 1440 \end{aligned}$$
The sum of the angles in a decagon is 1440°.

c. $a = (n-2) \times 180$
$= (12-2) \times 180$
$= 10 \times 180$
$= 1800$

The sum of the angles in a dodecagon is 1800°.

4. a. The sum of the angles in a triangle is 180°.

$x = 30°$

b. The sum of the angles in a quadrilateral is 360°.

$y = 70°$

c. The sum of the angles in a hexagon is 720°.

$a = 150°$

d. The sum of the angles in a pentagon is 540°.

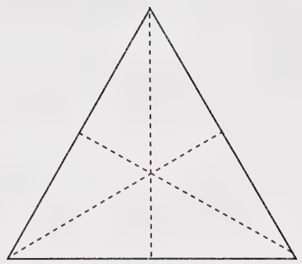
$b = 135°$

Section 4: Practice Activity 1

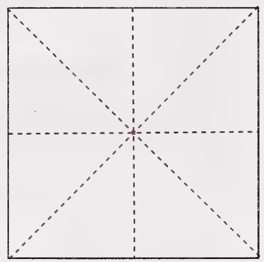
1. Each angle of the regular triangle is 60°.
Each angle of the regular quadrilateral is 90°.
Each angle of the regular pentagon is 108°.
Each angle of the regular hexagon is 120°.
Each angle of the regular octagon is 135°.
Each angle of the regular decagon is 144°.

2. a. Yes

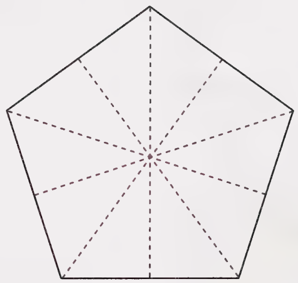
b. A regular triangle has three lines of symmetry.



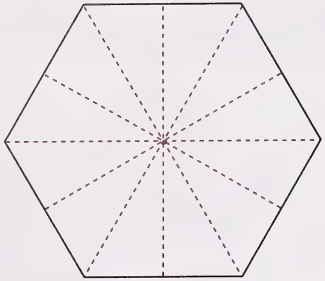
A regular quadrilateral has four lines of symmetry.



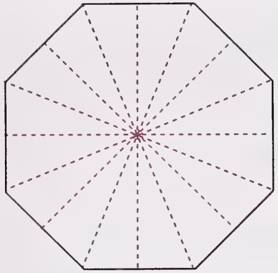
A regular pentagon has five lines of symmetry.



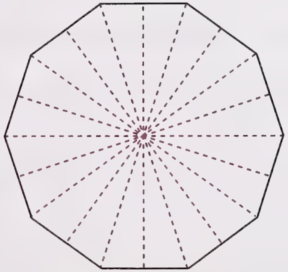
A regular hexagon has six lines of symmetry.



A regular octagon has eight lines of symmetry.



A regular decagon has 10 lines of symmetry.



c. $\ell = n$

3. a. A regular triangle has turn symmetry of order 3.
A regular quadrilateral has turn symmetry of order 4.
A regular pentagon has turn symmetry of order 5.
A regular hexagon has turn symmetry of order 6.
A regular octagon has a turn order of 8.
A regular decagon has a turn order of 10.

b. $o = n$

4. a. Yes, it has seven lines of symmetry.
b. Yes, it has a turn order of 7.
c. Yes, it has nine lines of symmetry.
d. Yes, it has a turn order of 9.

5. Yes, each regular polygon can be circumscribed.

Section 4: Practice Activity 2

1. square or regular quadrilateral

2. regular octagon

3. equilateral triangle or regular triangle

Section 4: Practice Activity 3

1. self corrected using a ruler and protractor

2. a. $360 \div 5 = 72$

Each central angle for a regular pentagon is 72°.

b. $360 \div 6 = 60$

Each central angle for a regular hexagon is 60°.

c. $360 \div 9 = 40$

Each central angle for a regular nonagon is 40°.

d. $360 \div 10 = 36$

Each central angle for a regular decagon is 36°.

3. self corrected using a ruler and protractor

4. The radius and each of the sides have the same measure.

5. a regular hexagon

6. self corrected using a ruler and protractor

7. Yes

8. a. 5 b. 7 c. 8 d. 10

Section 4: Practice Activity 4

1. Sample programs are given. Answers will vary.

a.	b.	c.	d.	e.
FD 30	FD 30	FD 30	FD 30	FD 30
RT 120	RT 90	RT 72	RT 60	RT 45
FD 30	FD 30	FD 30	FD 30	FD 30
RT 120	RT 90	RT 72	RT 60	RT 45
FD 30	FD 30	FD 30	FD 30	FD 30
RT 120	RT 90	RT 72	RT 60	RT 45
	FD 30	FD 30	FD 30	FD 30
	RT 90	RT 72	RT 60	RT 45
		FD 30	FD 30	FD 30
		RT 72	RT 60	RT 45
			FD 30	FD 30
			RT 60	RT 45
				FD 30
				RT 45
				FD 30
				RT 45

2. a. Repeat 3[FD 30 RT 120] b. Repeat 4[FD 30 RT 90]
c. Repeat 5[FD 30 RT 72] d. Repeat 6[FD 30 RT 60]
e. Repeat 8[FD 30 RT 45]

Section 5: Practice Activity 1

self corrected

Section 5: Practice Activity 2

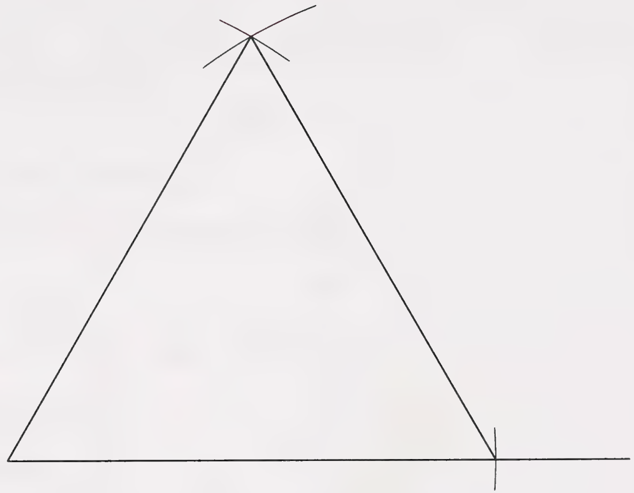
self corrected

Section 5: Practice Activity 3

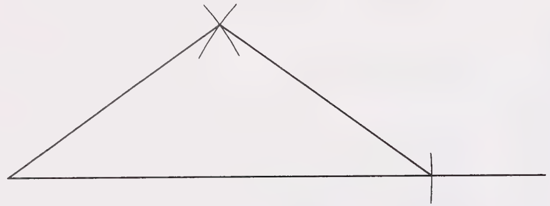
self corrected

Section 5: Practice Activity 4

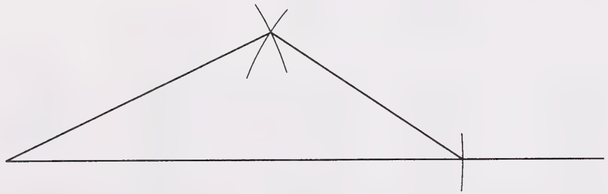
1. a.



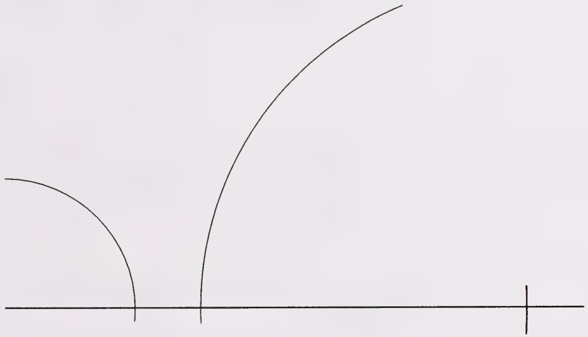
b.



c.



d. No, a triangle cannot be constructed with sides of these lengths. The sides won't meet to form a triangle.



The measure of the longest side of a triangle is always greater than the sum of the lengths of the other two sides.

$8 \not> 2 + 5$

2. self corrected

3. self corrected

Section 6: Practice Activity 1

self corrected

Section 6: Practice Activity 2

1. c. Each angle is 90°.

The rest of the activity is self corrected.

Section 6: Practice Activity 3

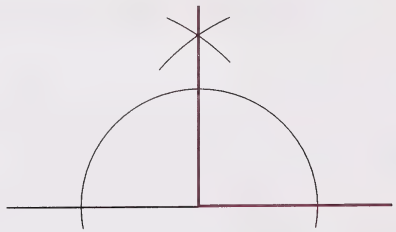
1. c. Each angle is 90°.

The rest of the activity is self corrected.

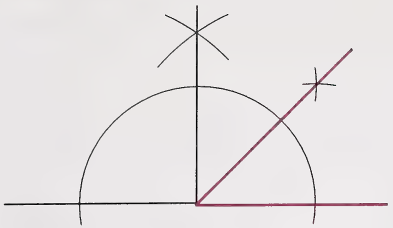
Section 6: Practice Activity 4

This activity is self-checked, but possible constructions are given.

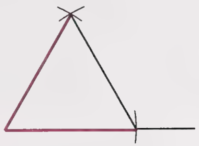
1. a. $\frac{1}{2}$ of $180° = 90°$



b. $\frac{1}{2}$ of $90° = 45°$

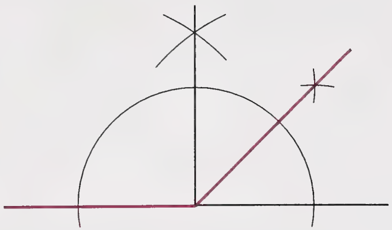


c. Each angle of equilateral triangle is 60°.

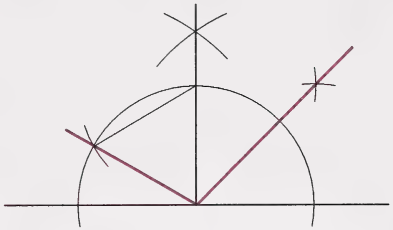


d. $\frac{1}{2}$ of $60° = 30°$

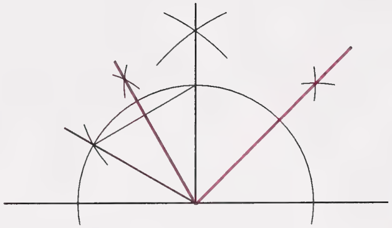
2. a. $90° + 45° = 135°$



b. $45° + 60° = 105°$



c. $45° + 30° = 75°$



Section 7: Practice Activity 1

self corrected

Section 7: Practice Activity 2

self corrected

Section 8: Practice Activity 1

1. a. $P = 2\ell + 2w$
   $= 2 \times 5.1 + 2 \times 7.2$
   $= 10.2 + 14.4$
   $= 24.6$

   The rectangle has a perimeter of 24.6 m.

   b. $P = 2\ell + 2w$
   $= 2 \times 15.6 + 2 \times 11.4$
   $= 31.2 + 22.8$
   $= 54$

   The perimeter of the parallelogram is 54 cm.

   c. $P = n \times s$
   $= 3 \times 47$
   $= 141$

   The perimeter of the triangle is 141 cm.

   1 m = 100 cm
   So, 141 cm = 1.41 m

   The perimeter of the triangle is 1.41 m.

   d. $P = n \times s$
   $= 6 \times 7.4$
   $= 44.4$

   The perimeter of the hexagon is 44.4 cm.

2. a. $C = \pi d$
   $\doteq 3.14 \times 18$
   $\doteq 56.52$

   The circumference of the circle is about 56.52 mm.

   b. $C = \pi d$
   $\doteq 3.14 \times 12.2$
   $\doteq 38.3$

   $r = 6.1$ m
   So, $d = 12.2$ m

   The circumference of the circle is about 38.3 m.

3. $P = n \times s$
   $= 5 \times 302$
   $= 1510$

   The minimum distance you would walk is 1510 m.

   1 km = 1000 m
   So, 1510 m = 1.51 km

   The minimum distance you would walk is 1.51 km.

4.

1 m = 100 cm

So, 60 cm = 0.6 m

$P = 2\ell + 2w$

$= 2 \times 5.1 + 2 \times 0.6$

$= 10.2 + 1.2$

$= 11.4$

The perimeter of the flower bed is 11.4 m.

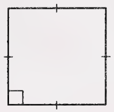
Section 8: Practice Activity 2

1. TAKETHESHORTESTROOT
(Take the shortest root (route).)

2. $P = 4s$

$= 4 \times 820$

$= 3280$



The perimeter of the square pasture is 3280 m.

1 km = 1000 m

So, 3280 m = 3.28 km

The perimeter of the square pasture is 3.28 km.

$3.28 \times 3 = 9.84$

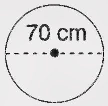
The farmer would need 9.84 km of barbed wire.

3. a. In one turn the wheel travels a distance equal to the circumference of the wheel.

$C = \pi d$

$\doteq 3.14 \times 70$

$\doteq 219.8$



Each wheel travels about 219.8 cm in one turn.

b.

1 km = 1000 m

1 m = 100 cm

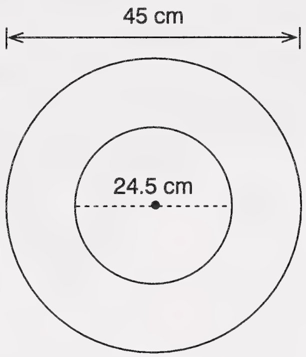
So, 1 km = 100 000 cm

and 6 km = 600 000 cm

$600\,000 \div 70 \doteq 8571.4$

Each wheel made about 8571.4 turns in the nonstop trip.

4.



a. $C = \pi d$

$\doteq 3.14 \times 45$

$\doteq 141.3$

The circumference of the hoop is about 141.3 cm.

$C = \pi d$

$\doteq 3.14 \times 24.5$

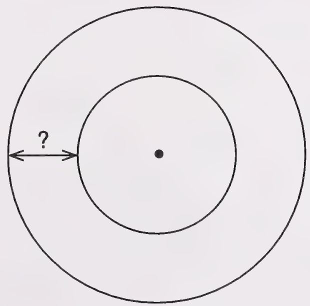
$\doteq 76.9$

The circumference of the basketball is about 76.9 cm.

$141.3 - 76.9 = 64.4$

The circumference of the hoop is about 64.4 cm greater.

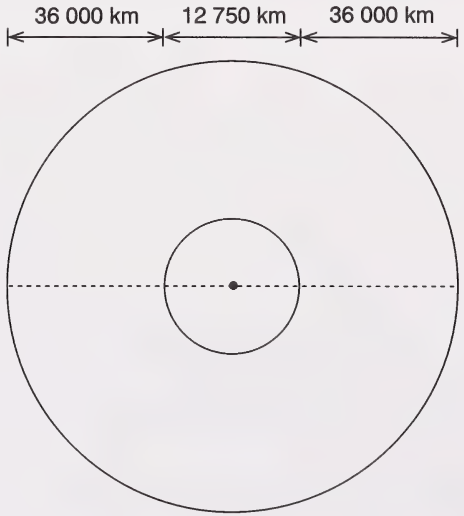
b.



$$\frac{45 - 24.5}{2} = \frac{20.5}{2}$$
$$= 10.25$$

The distance between the ball and the hoop is 10.25 cm.

5.



a. $C = \pi d$

$\doteq 3.14 \times 12\,750$

$\doteq 40\,035$

The circumference of the earth is about 40 035 km.

b. $36\,000 + 12\,750 + 36\,000 = 84\,750$

The diameter of the orbit is about 84 750 km.

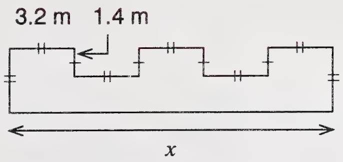
$C = \pi d$

$\doteq 3.14 \times 84\,750$

$\doteq 266\,115$

The satellite travels about 266 115 km in one orbit.

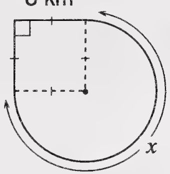
Section 8: Practice Activity 3

1. 

$$x = 5 \times 3.2$$
$$= 16$$

$$P = 7 \times 3.2 + 4 \times 1.4 + 16$$
$$= 22.4 + 5.6 + 16$$
$$= 44$$

The perimeter is 44 m.

2. 

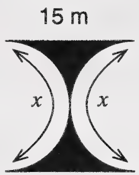
$x = \frac{3}{4}$ of the circumference a circle with a radius of 8 cm.

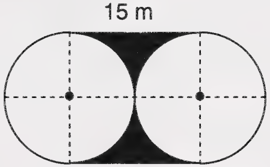
$$C = \frac{3}{4} \times \pi d$$
$$\doteq \frac{3}{\cancel{4}_{1}} \times 3.14 \times \cancel{16}^{4}$$
$$\doteq 37.68$$

$r = 8$ cm
So, $d = 16$ cm

$$P = 8 + 8 + 37.68$$
$$= 16 + 37.68$$
$$= 53.68$$

The perimeter of the figure is about 53.68 cm.

3. 



$x = \frac{1}{2}$ of the circumference of a circle with radius of 7.5 m.

$$C = \pi d$$
$$\doteq \frac{1}{2} \times 3.14 \times 15$$
$$\doteq 23.55$$

$$P = (2 \times 15) + (2 \times 23.55)$$
$$= 30 + 47.1$$
$$= 77.1$$

The circumference of the figure is 77.1 cm.

Section 9: Practice Activity 1

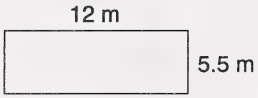
1. a. $A = b \times h$
$$= 6.8 \times 4$$
$$= 27.2$$

The area of the rectangle is 27.2 cm^2.

b. $A = b \times h$
$$= 17 \times 40$$
$$= 680$$

The area of the rectangle is 680 mm^2.

2. $A = b \times h$
$$= 12 \times 5.5$$
$$= 66$$



The area of the pad would be 66 m^2.

3. $A = b \times h$

$= 7.5 \times 3.4$

$= 25.5$

The area of the room is 25.5 m.

$32.95 \times 25.5 \doteq 840.23$

The cost would be $840.23.

Section 9: Practice Activity 2

1. a. $A = b \times h$

$= 6 \times 8$

$= 48$

The area of the parallelogram is 48 cm^2.

b. $A = b \times h$

$= 7 \times 5$

$= 35$

The area of the parallelogram is 35 cm^2.

2. $A = b \times h$

$= 3.2 \times 1.7$

$= 5.44$

The area of one flight of a dart is 5.44 cm^2.

3. $A = b \times h$

$= 16 \times 2.5$

$= 40$

The area of the sidewalk is 40 m^2.

Section 9: Practice Activity 3

1. a. $A = \frac{b \times h}{2}$

$= \frac{9 \times 12}{2}$

$= \frac{108}{2}$

$= 54$

The area of the triangle is 54 cm^2.

b. $A = \frac{b \times h}{2}$

$= \frac{5 \times 9}{2}$

$= \frac{45}{2}$

$= 22.5$

The area of the triangle is 22.5 m^2.

c. $A = \frac{b \times h}{2}$

$= \frac{15 \times 6}{2}$

$= \frac{90}{2}$

$= 45$

The area of the triangle is 45 cm^2.

2. $A = \frac{b \times h}{2}$

$= \frac{60 \times 15}{a}$

$= \frac{900}{2}$

$= 450$

The area of the pennant is 450 cm^2.

3. $A = \frac{b \times h}{2}$

$= \frac{6.5 \times 1.5}{2}$

$\doteq 4.9$

The area of the gable is about 4.9 m^2.

4. $A = \frac{b \times h}{2}$

$= \frac{6 \times 3}{2}$

$= 9$

The area of the sail is 9 m^2.

Section 9: Practice Activity 4

1. a.
$$\begin{aligned} A &= \frac{(b_1+b_2)\times h}{2} \\ &= \frac{(12+34)}{2}\times 11 \\ &= \frac{46\times 11}{2} \\ &= \frac{506}{2} \\ &= 253 \end{aligned}$$

The area of the trapezoid is 253 cm^2.

b.
$$\begin{aligned} A &= \frac{(b_1+b_2)\times h}{2} \\ &= \frac{(6.1+4.7)}{2}\times 4.7 \\ &= \frac{(10.8)}{2}\times 4.7 \\ &= \frac{50.76}{2} \\ &= 25.38 \end{aligned}$$

The area of the trapezoid is 25.38 cm^2.

2.
$$\begin{aligned} A &= \frac{(b_1+b_2)\times h}{2} \\ &= \frac{(40+30)\times 20}{2} \\ &= \frac{70\times 20}{2} \\ &= \frac{1400}{2} \\ &= 700 \end{aligned}$$

The area of the side of the wheel barrow is 700 cm^2.

3.
$$\begin{aligned} A &= \frac{(b_1+b_2)\times h}{2} \\ &= \frac{(2.5+3.5)\times 2}{2} \\ &= \frac{6\times 2}{2} \\ &= 6 \end{aligned}$$

The area of the side of the tent is 6 m^2.

Section 9: Practice Activity 5

1. a.
$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times 2^2 \\ &\doteq 3.14\times 4 \\ &\doteq 12.6 \end{aligned}$$

The area of the circle is about 12.6 m^2.

b.
$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times 14^2 \\ &\doteq 3.14\times 196 \\ &\doteq 615.4 \end{aligned}$$

$d = 28$ m
So, $r = 14$ m

The area of the circle is about 615.4 mm^2.

2.
$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times 10^2 \\ &\doteq 3.14\times 100 \\ &\doteq 314 \end{aligned}$$

The area would be about 314 m^2.

3.
$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times 25^2 \\ &\doteq 3.14\times 625 \\ &\doteq 1962.5 \end{aligned}$$

The area would be about 1962.5 m^2.

4.
$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times (7.5)^2 \\ &\doteq 3.14\times 56.25 \\ &\doteq 176.625 \end{aligned}$$

The area of a 15-cm diameter pizza is about 176.6 cm^2. The area of two 15-cm pizzas is about 353.25 cm^2.

$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\times (12.5)^2 \\ &\doteq 3.14\times 156.25 \\ &\doteq 490.625 \end{aligned}$$

The area of the 25-cm diameter pizza is 490.625 cm^2.

$490.625 > 353.25$

So, the purchase of the 25-cm pizza is the better deal.

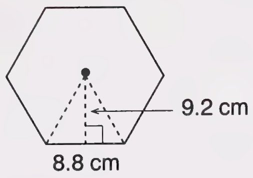
Section 9: Practice Activity 6

1. $A = \frac{b \times h}{2}$

$= \frac{8.8 \times 9.2}{2}$

$= \frac{80.96}{2}$

$= 40.48$



The area of one triangle is 40.48 cm^2.

$A = 6 \times 40.48$

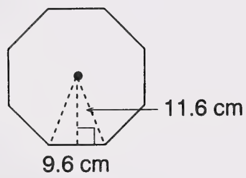
$\doteq 242.9$

The area of the hexagon is about 242.9 cm^2.

2. $A = \frac{b \times h}{2}$

$= \frac{9.6 \times 11.6}{2}$

$\doteq 55.7$



The area of one triangle is about 55.7 cm^2.

$A \doteq 8 \times 55.7$

$\doteq 445.4$

The area of the octagon is about 445.4 cm^2.

Section 9: Practice Activity 7

1. a. $A = \frac{n \times s \times a}{2}$

$= \frac{5 \times 1.6 \times 1.1}{2}$

$= 4.4$

The area of the pentagon is 4.4 km^2.

b. $A = \frac{n \times s \times a}{2}$

$= \frac{8 \times 45 \times 54}{2}$

$= 9720$

The area of the octagon is 9720 cm^2.

c. $A = \frac{n \times s \times a}{2}$

$= \frac{6 \times 5.6 \times 4.8}{2}$

$= 80.64$

The area of the hexagon is 80.64 cm^2.

d. $A = \frac{n \times s \times a}{2}$

$= \frac{10 \times 18 \times 28}{2}$

$= 2520$

The area of the decagon is 2520 mm^2.

1 cm^2 = 100 mm^2

So, 2520 mm^2 = 25.2 cm^2

The area of the decagon is 25.2 cm^2.

2. $A = \frac{n \times s \times a}{2}$

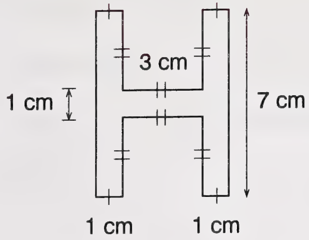
$= \frac{8 \times 16.5 \times 20}{2}$

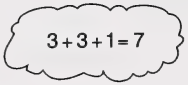
$= 1320$

The area of the stop sign is 1320 cm^2.

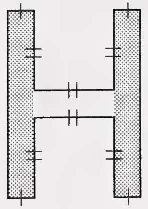
Section 9: Practice Activity 8

1. a.





Area of two congruent rectangles

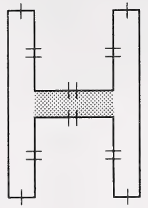


$A = b \times h$
$= 1 \times 7$
$= 7$

Each rectangle has an area of 7 cm^2.

Two rectangles have an area of 14 cm^2.

Area of other rectangle



$A = b \times h$
$= 3 \times 1$
$= 3$

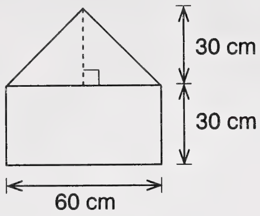
The area of the other rectangle is 3 cm^2.

Area of total figure

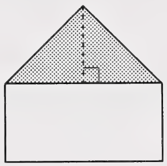
$14 + 3 = 17$

The total area of the figure is 17 cm^2.

b.



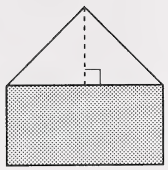
Area of triangle



$A = \frac{b \times h}{2}$
$= \frac{60 \times 30}{2}$
$= 900$

The area of the triangle is 900 cm^2.

Area of rectangle



$A = b \times h$
$= 60 \times 30$
$= 1800$

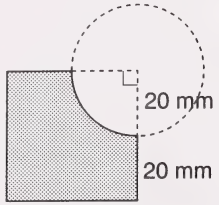
The area of the rectangle is 1800 cm^2.

Total area of the figure

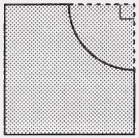
$900 + 1800 = 2700$

The area of the entire figure is 2700 cm^2.

c.



Area of the square

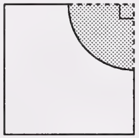


$$A = b \times h$$
$$= 40 \times 40$$
$$= 1600$$

The area of the square is 1600 mm^2.

Area of circular part

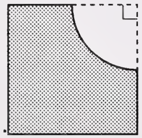
The circular part is one-fourth of a circle with a radius of 20 mm.



$$A = \frac{1}{4} \times \pi r^2$$
$$\doteq \frac{1}{4} \times 3.14 \times 20^2$$
$$\doteq \frac{1}{4} \times 3.14 \times 400$$
$$\doteq 314$$

The area of the circular part is about 314 mm^2.

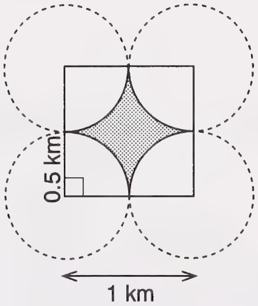
Shaded area



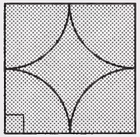
$$1600 - 314 = 1286$$

The area of the shaded part is about 1286 mm^2.

d.



Area of the square

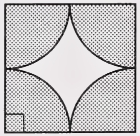


$$A = b \times h$$
$$= 1 \times 1$$
$$= 1$$

The area of the square is 1 km^2.

Area of circular parts

There are four circular parts. Each circular part is one-fourth of circle with a radius of 0.5 km.



$$A = \overset{1}{\cancel{4}} \times \frac{1}{\underset{1}{\cancel{4}}} \pi r^2$$
$$\doteq 3.14 \times (0.5)^2$$
$$\doteq 3.14 \times 0.25$$
$$\doteq 0.785$$

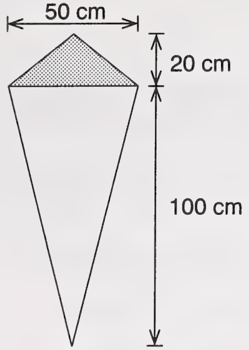
The area of the four circular parts is about 0.785 km^2.

Shaded area

$1 - 0.785 = 0.215$

The area of the shaded part is about 0.215 km^2.

2.

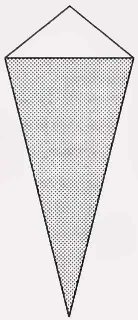


Area of one triangle

$$A = \frac{b \times h}{2}$$
$$= \frac{50 \times 20}{2}$$
$$= \frac{1000}{2}$$
$$= 500$$

The area is 500 cm^2.

Area of second triangle



$$A = \frac{b \times h}{2}$$
$$= \frac{50 \times 100}{2}$$
$$= 2500$$

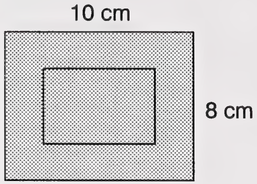
The area of the second triangle is 2500 cm^2.

Area of two triangles

$2500 + 500 = 3000$

The area of the kite is 3000 cm^2.

3. Area of outside rectangle

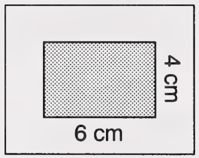


$A = b \times h$
$= 10 \times 8$
$= 80$

The area of the outside rectangle is 80 cm^2.

Area of inside rectangle

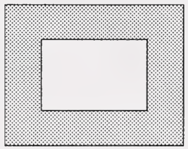
$10 - 4 = 6$ $\quad$ $8 - 4 = 4$



$A = b \times h$
$= 6 \times 4$
$= 24$

The area of the inside rectangle is 24 cm^2.

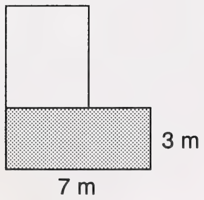
Area of frame



$80 - 24 = 56$

The area of the frame is 56 cm^2.

4. a. Area of one room

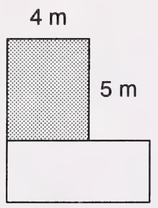


$A = b \times h$
$= 7 \times 3$
$= 21$

The area of one room is 21 m^2.

Area of second room

$8 - 3 = 5$



$A = b \times h$
$= 4 \times 5$
$= 20$

The area of the second room is 20 m^2.

Total area

$20 + 21 = 41$

The area of both rooms is 41 m^2.

b. Cost of carpeting

$41 \times 34.24 = 1403.84$

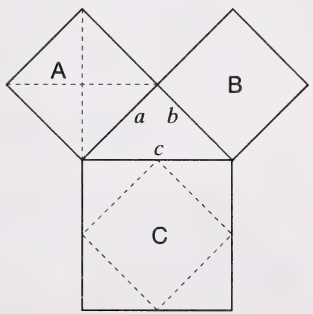
The cost is $1403.84.

Section 10: Practice Activity 1

1. The area of square A plus the area of square B equals the area of square C.

   or

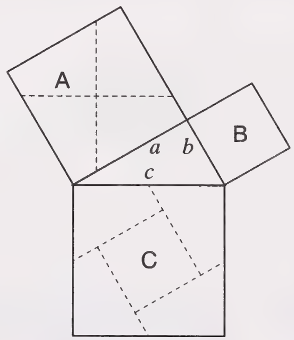
   $a^2 + b^2 = c^2$



2. The area of square A plus the area of square B equals the area of square C.

   or

   $a^2 + b^2 = c^2$



Section 10: Practice Activity 2

1. a.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 35^2 + 12^2 &= c^2 \\ 1225 + 144 &= c^2 \\ c^2 &= 1369 \\ c &= 37 \end{aligned}$$

The hypotenuse is 37 cm.

b.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 55^2 + b^2 &= 73^2 \\ 3025 + b^2 &= 5329 \\ b^2 &= 2304 \\ b &= 48 \end{aligned}$$

The leg is 48 m.

c.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + 16^2 &= c^2 \\ 900 + 256 &= c^2 \\ c^2 &= 1156 \\ c &= 34 \end{aligned}$$

The hypotenuse is 34 mm.

d.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ (6.3)^2 + a^2 &= (6.5)^2 \\ 39.69 + a^2 &= 42.25 \\ a^2 &= 2.56 \\ a &\doteq 1.6 \end{aligned}$$

The leg is about 1.6 m.

e.

$$a^2 + b^2 = c^2$$
$$(5.2)^2 + (2.0)^2 = c^2$$
$$27.04 + 4 = c^2$$
$$c^2 = 31.04$$
$$c \doteq 5.57$$

The hypotenuse is about 5.57 km.

f.

$$a^2 + b^2 = c^2$$
$$a^2 + (2.0)^2 = (2.9)^2$$
$$a^2 + 4 = 8.41$$
$$a^2 = 4.41$$
$$a = 2.1$$

The leg is 2.1 cm.

2.

$$a^2 + b^2 = c^2$$
$$a^2 + 35^2 = 50^2$$
$$a^2 + 1225 = 2500$$
$$a^2 = 1275$$
$$a \doteq 35.7$$

The kite is about 35.7 m above Zorna.

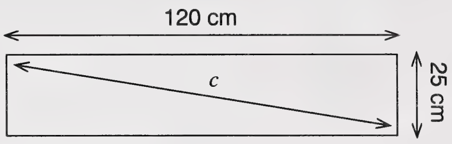
3.

$$a^2 + b^2 = c^2$$
$$(2.4)^2 + b^2 = 3.0^2$$
$$5.76 + b^2 = 9$$
$$b^2 = 3.24$$
$$b = 1.8$$

It is 1.8 km across the lake.

4.

$$a^2 + b^2 = c^2$$
$$120^2 + 25^2 = c^2$$
$$14\,400 + 625 = c^2$$
$$c^2 = 15\,025$$
$$c \doteq 122.6$$



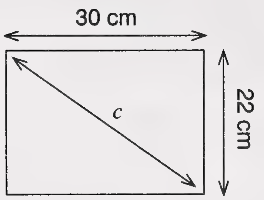
The longest ski pole that could fit in the box is about 122.6 cm.

1 m = 100 cm
So, 122.6 cm = 1.226 m

The longest ski pole that could fit in the box is 1.226 m.

5.

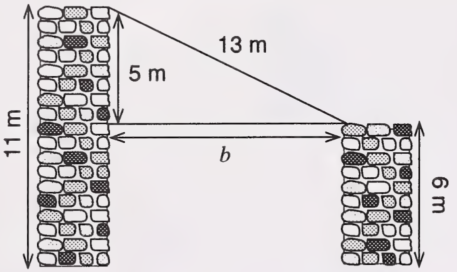
$$a^2 + b^2 = c^2$$
$$30^2 + 22^2 = c^2$$
$$900 + 484 = c^2$$
$$c^2 = 1384$$
$$c \doteq 37.2$$



The diagonal measure of the television screen is 37.2 cm.

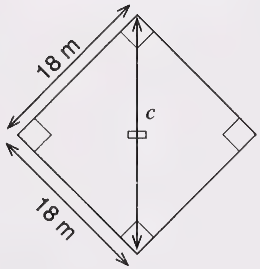
6. $a = 11 - 6$
$= 5$

$a^2 + b^2 = c^2$
$5^2 + b^2 = 13^2$
$25 + b^2 = 169$
$b^2 = 144$
$b = 12$



The distance between the walls is 12 m.

6. $a^2 + b^2 = c^2$
$18^2 + 18^2 = c^2$
$324 + 324 = c^2$
$c^2 = 648$
$c \doteq 25.5$



The player must throw the ball 25.5 m.

Section 11: Practice Activity 1

1. The name of each prism is related to the shape of the base.

   - The rectangular prism has congruent rectangles for bases.
   - The triangular prism has congruent triangles for bases.
   - The pentagonal prism has congruent pentagons for bases.
   - The hexagonal prism has congruent hexagons for bases.

2.

Prism	Number of Bases	Number of Lateral Faces	Number of Edges	Number of Vertices
Rectangular Prism	2	4	12	8
Triangular Prism	2	3	9	6
Pentagonal Prism	2	5	15	10
Hexagonal Prism	2	6	18	12

3. The prisms with bases that are regular polygons have all the lateral faces congruent.

Section 11: Practice Activity 3

1. **a.** The length of the rectangle is equal to the circumference of the circle.

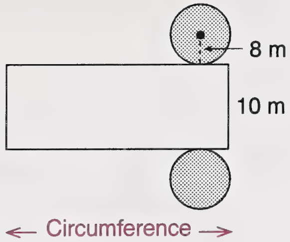
   b. The width of the rectangle is equal to the height of the cylinder.

2. **a.** pentagonal prism
   b. rectangular prism
   c. triangular prism
   d. cylinder

3. **a.** pentagonal prism
   b. rectangular prism or cube
   c. cylinder

Section 12: Practice Activity 1

1. a. **Area of bases**



$A = \pi r^2$

$\doteq 3.14 \times 8^2$

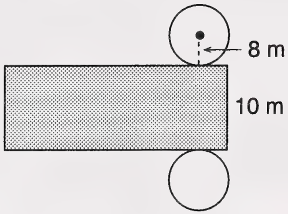
$\doteq 3.14 \times 64$

$\doteq 200.96$

The area of one base is about 200.96 m^2.

The area of two bases is about 401.92 m^2.

Area of rectangle



$C = \pi d$

$\doteq 3.14 \times 16$

$\doteq 50.24$

$A = b \times h$

$\doteq 50.24 \times 10$

$\doteq 502.4$

The area of the rectangle is about 502.4 m^2.

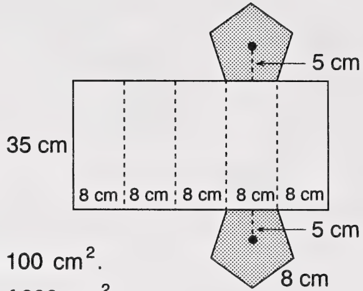
Surface area of cylinder

$SA = 401.92 + 502.4$

$= 904.32$

The surface area of the cylinder is 904.32 m^2.

b. **Area of bases**



$A = \dfrac{n \times s \times a}{2}$

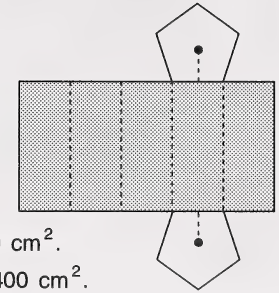
$= \dfrac{5 \times \cancel{8}^{4} \times 5}{\cancel{2}_{1}}$

$= 100$

Each base has an area of 100 cm^2.

Two bases have an area of 200 cm^2.

Area of Rectangles



$A = b \times h$

$= 8 \times 35$

$= 280$

Each rectangle has an area of 280 cm^2.

Five rectangles have an area of 1400 cm^2.

Surface area of pentagonal prism

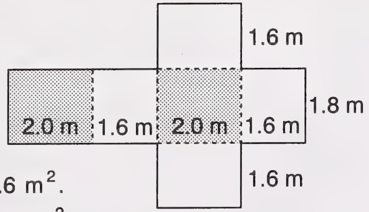
$SA = 200 + 1400$

$= 1600$

The surface area of the pentagonal prism is 1600 cm^2.

c. **Area of bases**

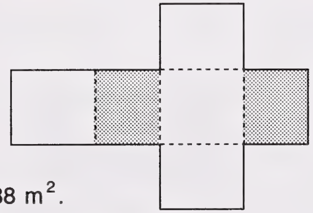
$A = b \times h$
$= 2 \times 1.8$
$= 3.6$



The area of one base is 3.6 m^2.
The area of two bases is 7.2 m^2.

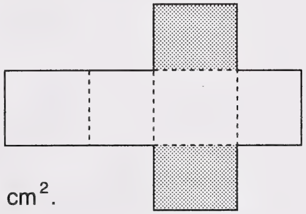
Area of rectangles

$A = b \times h$
$= 1.8 \times 1.6$
$= 2.88$



The area of one rectangle is 2.88 m^2.
The area of two congruent rectangles is 5.76 m^2.

$A = b \times h$
$= 2.0 \times 1.6$
$= 3.2$



The area of one rectangle is 3.2 cm^2.
The area of two congruent rectangles is 6.4 cm^2.

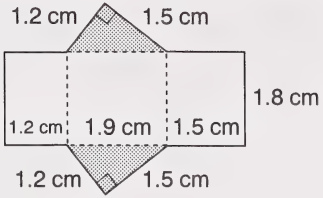
Surface area of rectangular prism

$SA = 7.2 + 5.76 + 6.4$
$= 19.36$

The surface area of the rectangular prism is 19.36 cm^2.

d. **Area of bases**

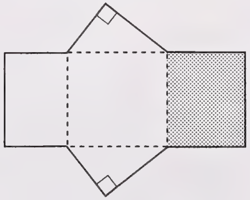
$A = \frac{b \times h}{2}$
$= \frac{1.2 \times 1.5}{2}$
$= 0.9$



One base has an area of 0.9 cm^2.
Two bases have an area of 1.8 cm^2.

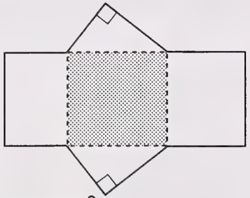
Area of rectangles

$A = b \times h$
$= 1.8 \times 1.5$
$= 2.7$



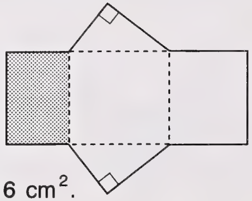
One rectangle has an area of 2.7 cm^2.

$A = b \times h$
$= 1.9 \times 1.8$
$= 3.42$



The second rectangle has an area of 3.42 cm^2.

$A = b \times h$
$= 1.2 \times 1.8$
$= 2.16$



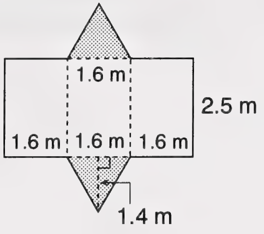
The third rectangle has an area of 2.16 cm^2.

Surface area of triangular prism

$$SA = 1.8 + 2.7 + 3.42 + 2.16$$
$$= 10.08$$

The surface area of the triangular prism is 10.08 cm^2.

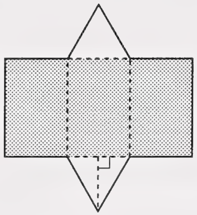
2. **Area of bases**



$$A = \frac{b \times h}{2}$$
$$= \frac{1.6 \times 1.4}{2}$$
$$= 1.12$$

The area of one triangle is 1.12 m^2.
The area of two triangles is 2.24 m^2.

Area of rectangles



$$A = b \times h$$
$$= 1.6 \times 2.5$$
$$= 4$$

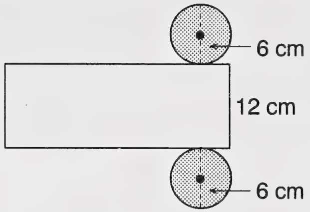
The area of one rectangle is 4 m^2.
The area of three rectangles is 12 m^2.

Surface area of tent

$$SA = 2.24 + 12$$
$$= 14.24$$

The surface area of the tent is 14.24 m^2. So, 14.24 m^2 of canvas was used, assuming there was no wastage.

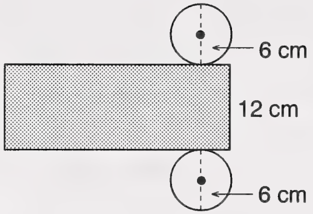
3. **The area of the base**



$$A = \pi r^2$$
$$\doteq 3.14 \times 3^2$$
$$\doteq 3.14 \times 9$$
$$\doteq 28.26$$

The area of one base is 28.26 cm^2.
The area of two bases is 56.52 cm^2.

Area of rectangle



$$c = \pi d$$
$$\doteq 3.14 \times 6$$
$$\doteq 18.84$$

$$A = b \times h$$
$$= 18.84 \times 12$$
$$= 226.08$$

The area of the rectangle is 226.08 cm^2.

Surface area of cylinder

$$SA = 56.52 + 226.08$$
$$= 282.6$$

The surface area of the cylinder is 282.6 cm^2. So, 282.6 cm^2 of metal sheet was used in the construction of the can, assuming there was no wastage.

Section 12: Practice Activity 2

1. a. $B = \frac{n \times s \times a}{2}$

$= \frac{6 \times 18 \times \cancel{12}^{6}}{\cancel{2}_{1}}$

$= 648$

The area of the hexagonal base is 648 cm².

$P = n \times s$

$= 6 \times 18$

$= 108$

The perimeter of the hexagonal base is 108 cm.

$SA = 2B + P \times H$

$= 2 \times 648 + 108 \times 30$

$= 1296 + 3240$

$= 4536$

The surface area of the hexagonal prism is 4536 cm².

b. $B = \pi r^2$

$\doteq 3.14 \times (0.5)^2$

$\doteq 0.785$

The area of the circular base is about 0.785 m².

$C = \pi d$

$\doteq 3.14 \times 1.0$

$\doteq 3.14$

The circumference of the circular base is about 3.14 m.

$SA = 2B + C \times H$

$\doteq 2 \times 0.785 + 3.14 \times 4.2$

$\doteq 1.57 + 13.19$

$\doteq 14.76$

The surface area of the cylinder is about 14.76 m².

c. $B = b \times h$

$= 7 \times 16$

$= 112$

The area of the rectangular base is 112 mm².

$P = 2\ell + 2w$

$= 2 \times 7 + 2 \times 16$

$= 14 + 32$

$= 46$

The perimeter of the rectangular base is 46 mm.

$SA = 2B + P \times H$

$= 2 \times 112 + 46 \times 5$

$= 224 + 230$

$= 454$

The surface area of the rectangular prism is 454 mm².

d. $B = \frac{b \times h}{2}$

$= \frac{12 \times 10}{2}$

$= 60$

The area of the triangular base is 60 m².

$P = 12 + 24 + 20$

$= 56$

The perimeter of the triangular base is 56 m.

$SA = 2B + P \times H$

$= 2 \times 60 + 56 \times 15$

$= 120 + 840$

$= 960$

The surface area of the triangular prism is 960 m².

2. $B = \frac{n \times s \times a}{2}$

$= \frac{5 \times \cancel{20}^{10} \times 15.75}{\cancel{2}_{1}}$

$= 787.5$

The area of the pentagonal base is 787.5 cm^2.

$P = n \times s$

$= 5 \times 20$

$= 100$

The perimeter of the pentagonal base is 100 cm.

$SA = 2B + P \times H$

$= 2 \times 787.5 + 100 \times 20$

$= 1575 + 2000$

$= 3575$

The surface area of the bird house is 3575 cm^2.

3. $B = \pi r^2$

$\doteq 3.14 \times 5^2$

$\doteq 3.14 \times 25$

$\doteq 78.5$

The area of the base is about 78.5 m^2.

$C = \pi d$

$\doteq 3.14 \times 10$

$\doteq 31.4$

The circumference of the circular base is about 31.4 m.

$SA = 2B + C \times H$

$\doteq 2 \times 78.5 + 3.14 \times 4$

$\doteq 157 + 12.56$

$\doteq 169.56$

The surface area of the oil tank is about 169.56 m^2.

Section 12: Practice Activity 3

1. $B = \pi r^2$

$\doteq 3.14 \times (1.8)^2$

$\doteq 3.14 \times 3.24$

$\doteq 10.2$

The area of the circular base is about 10.2 m^2.

$C = \pi d$

$\doteq 3.14 \times 3.6$

$\doteq 11.30$

The circumference of the circular base is about 11.3 m.

There is only one base.

$SA = B + C \times H$

$\doteq 10.2 + 11.3 \times 1$

$\doteq 10.2 + 11.3$

$\doteq 21.5$

The surface area of the pool is 21.5 m^2.

So, 21.5 m^2 of vinyl material will be required, assuming there is no wastage.

2. $P = 2\ell + 2w$

$= 2 \times 3.7 + 2 \times 4.4$

$= 7.4 + 8.8$

$= 16.2$

The perimeter of the rectangular base is 16.2 m.

$SA = P \times H$

$= 16.2 \times 2.5$

$= 40.5$

The surface area of the walls is 40.5 m^2.

3. $B = \frac{(b_1 + b_2) \times h}{2}$

$= \frac{(1.0 + 0.4) \times 0.3}{2}$

$= 0.21$

The area of the trapezoidal base is 0.21 m^2.

$P = 2 \times 0.5 + 0.4 + 1.0$

$= 1 + 0.4 + 1$

$= 2.4$

The perimeter of the trapezoidal base is 2.4 m.

There is only one base.

$SA = B + P \times H$

$= 0.21 + 2.4 \times 6$

$= 0.21 + 14.4$

$= 14.61$

The surface area of the water trough is 14.61 m^2.

So, if there is no wastage, 14.61 m^2 of steel is required to construct the water trough.

Section 13: Practice Activity 1

1. a. $B = \frac{b \times h}{2}$

$= \frac{3 \times 7}{2}$

$= 10.5$

The area of the triangular base is 10.5 m^2.

$V = B \times H$

$= 10.5 \times 12$

$= 126$

The volume of the triangular prism is 126 m^3.

b. $B = \pi r^2$

$\doteq 3.14 \times 8^2$

$\doteq 3.14 \times 64$

$\doteq 200.96$

The area of the circular base is about 200.96 cm^2.

$V = B \times H$

$\doteq 200.96 \times 15$

$\doteq 3014.4$

The volume of the cylinder is about 3014.4 m^3.

c. $B = \frac{n \times s \times a}{2}$

$= \frac{\overset{3}{\cancel{6}} \times 0.8 \times 0.7}{\underset{1}{\cancel{2}}}$

$= 1.68$

The area of the hexagonal base is 1.68 m^2.

$V = B \times H$

$= 1.68 \times 3.2$

$\doteq 5.4$

The volume of the hexagonal prism is about 5.4 m^3.

d. $B = b \times h$

$= 6 \times 20$

$= 120$

The area of the rectangular base is 120 mm^2.

$V = B \times H$

$= 120 \times 14$

$= 1680$

The volume of the rectangular prism is 1680 mm^3.

e. $B = b \times h$

$= 2.5 \times 2.5$

$= 6.25$

The area of the square base is 6.25 m^2.

$V = B \times H$

$= 6.25 \times 6.4$

$= 40$

The volume of the rectangular base is 40 m^3.

1. $B = \frac{b \times h}{2}$

$= \frac{12 \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}}}$

$= 48$

The area of the triangular base is 48 cm^2.

$V = B \times H$

$= 48 \times 22$

$= 1056$

The volume of the triangular prism is 1056 cm^3.

2. $B = \pi r^2$

$\doteq 3.14 \times 3^2$

$\doteq 3.14 \times 9$

$\doteq 28.26$

The area of the circular base is about 28.26 cm^2.

$V = B \times H$

$\doteq 28.26 \times 10$

$\doteq 282.6$

The volume of the cylindrical candle is about 282.6 cm^3.

So about 282.6 cm^3 of wax is needed.

3. $B = b \times h$

$= 3.2 \times 2.6$

$= 8.32$

The area of the rectangular base is 8.32 m^2.

$V = B \times H$

$= 8.32 \times 1.8$

$\doteq 14.98$

The volume of the box of the dump truck is about 14.98 m^3.

So, 14.98 m^3 of soil can be carried.

4. $B = \pi r^2$

$\doteq 3.14 \times (0.275)^2$

$= 3.14 \times 0.76$

$\doteq 0.24$

$V = B \times H$

$\doteq 0.24 \times 1$

$\doteq 0.24$

The volume of the plastic garbage can is about 0.24 m^3.

5. $B = b \times h$

$= 6.9 \times 5.8$

$= 40.02$

The area of the rectangular base is 40.02 m^2.

$V = B \times H$

$= 40.02 \times 0.2$ 20 cm = 0.2 m

$\doteq 8.00$

The volume of the patio is about 8 m^3.

$8 \times 15 = 120$

It will cost $120.

Section 13: Practice Activity 2

1. **Volume of carton**

$B = b \times h$

$= 48 \times 24$

$= 1152$

$V = B \times H$

$= 1152 \times 12$

$= 13\,824$

The volume of the carton is 13 824 cm^3.

Volume of cans

$$B = \pi r^2 \doteq 3.14 \times 4^2 \doteq 3.14 \times 16 \doteq 50.24$$

$$V = B \times H \doteq 50.24 \times 12 \doteq 602.88$$

The volume of one can is about 602.88 cm³.

The volume of 18 cans is about 10 851.84 cm^3.

Difference in volume

$13\,824 - 10\,851.84 = 2972.16$

The difference in the volumes is 2972.16 cm^3. So, there is 2972.16 cm^3 of space in the carton.

2. a. Answers will vary.

   b. The cylinder is easier to handle.

   c. The triangular prism looks larger.

   d. **Triangular prism**

$$B = \frac{b \times h}{2} = \frac{8 \times 8}{2} = 32$$

$$V = B \times H = 32 \times 18 = 576$$

The volume of the triangular prism is 576 cm^3.

Cylinder

$$B = \pi r^2 \doteq 3.14 \times 4^2 \doteq 3.14 \times 16 \doteq 50.24$$

$$V = B \times H = 50.24 \times 12 = 602.88$$

The volume of the cylinder is 602.88 cm^3.

Volume of rectangular prism

$$B = 8 \times 8 = 64$$

$$V = B \times H = 64 \times 12 = 768$$

The volume of the rectangular prism is 768 cm^3.

Comparison

$768 > 602.88$ and $768 > 576$

So, the volume of the rectangular prism is the greatest.

3. a. The customer probably will not notice the change.

b. Volume of original bar

$$B = b \times h$$
$$= 9 \times 5$$
$$= 45$$

$$V = B \times H$$
$$= 45 \times 1$$
$$= 45$$

The volume of the original bar is 45 cm^3.

Volume of new bar

$$B = b \times h$$
$$= 9 \times 5$$
$$= 45$$

$$V = B \times H$$
$$= 45 \times 0.9$$
$$= 40.5$$

The volume of the new bar is 40.5 cm^3.

Difference

$45 - 40.5 = 4.5$

The volume of the new bar is 4.5 cm^3 less than the volume of the original.

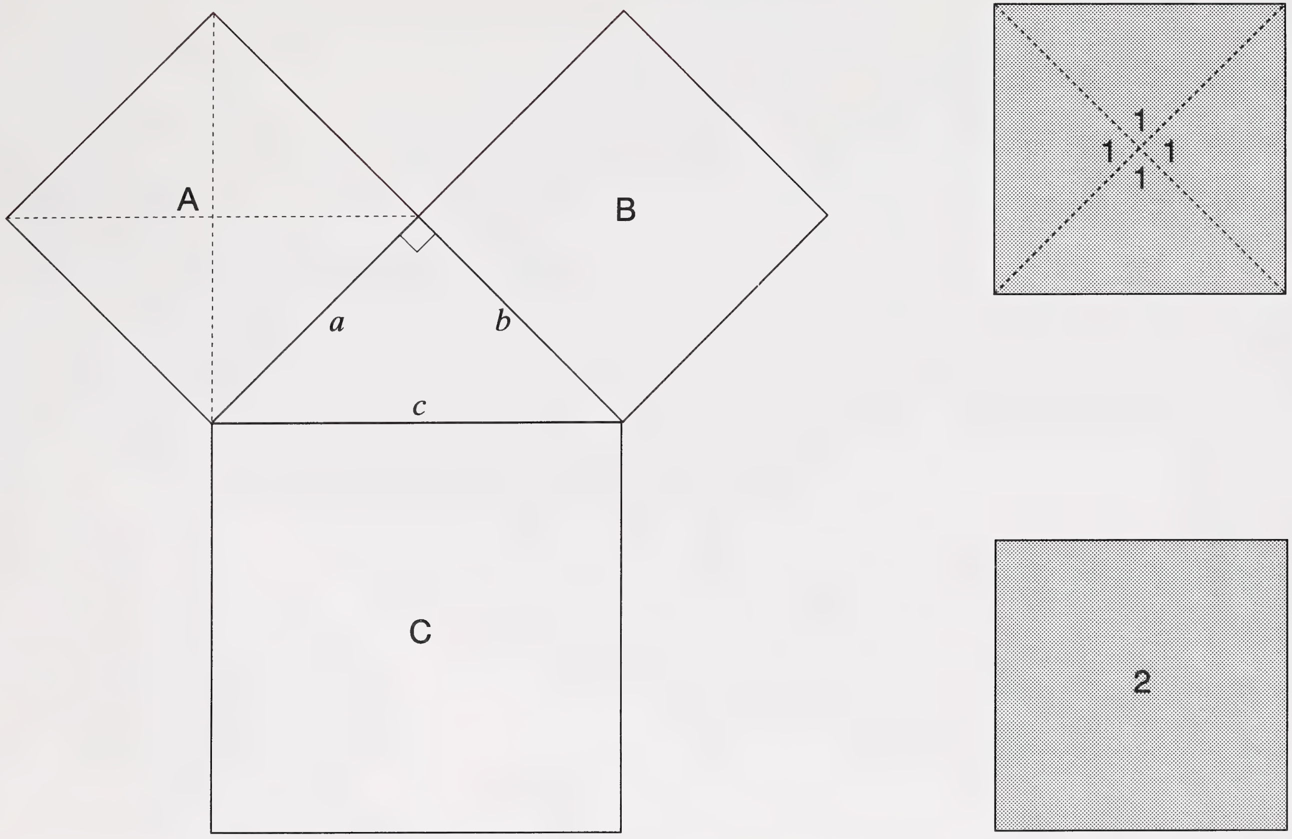
Percent decrease

$$\frac{4.5}{45} = \frac{45}{450}$$
$$= \frac{1}{10}$$
$$= 10\%$$

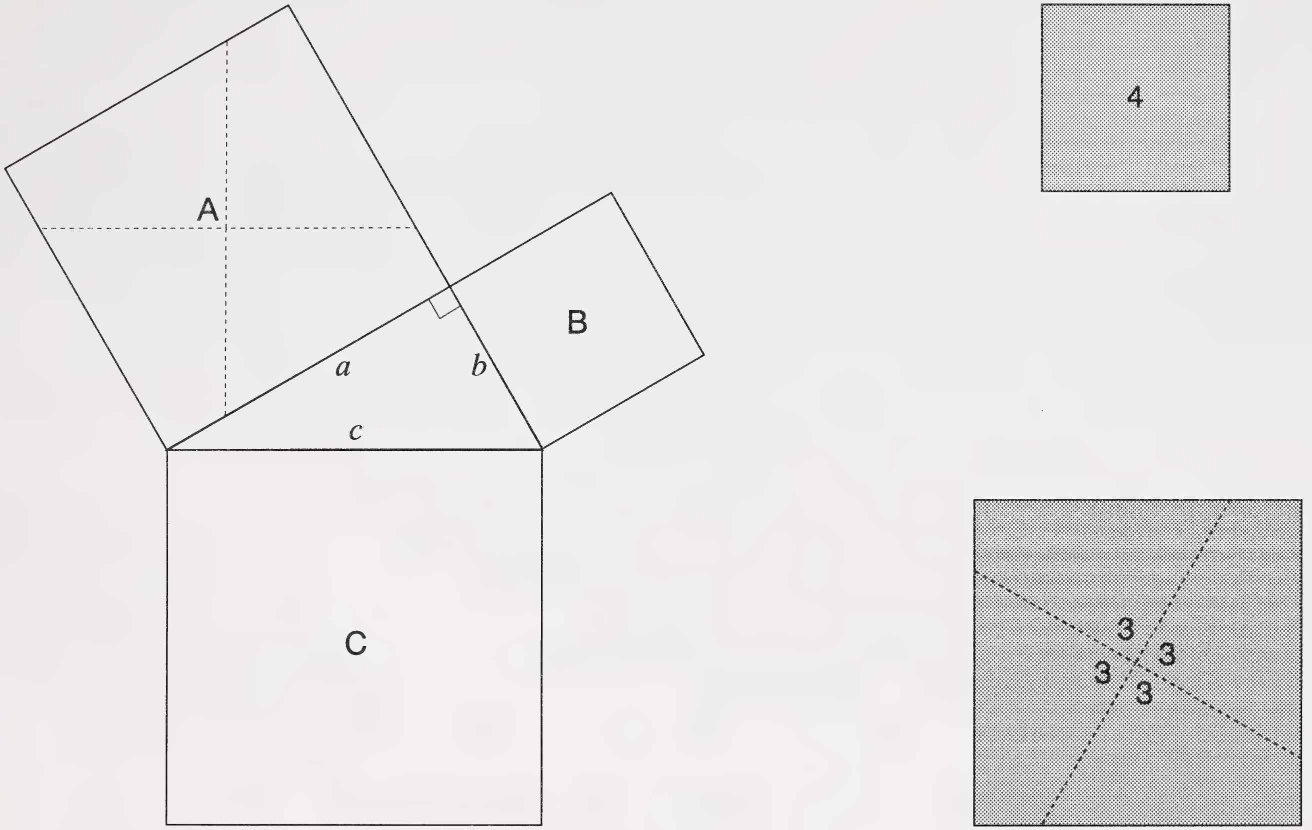
$\frac{\text{decrease in volume}}{\text{original volume}}$

The volume of the chocolate bar was reduced by 10%.

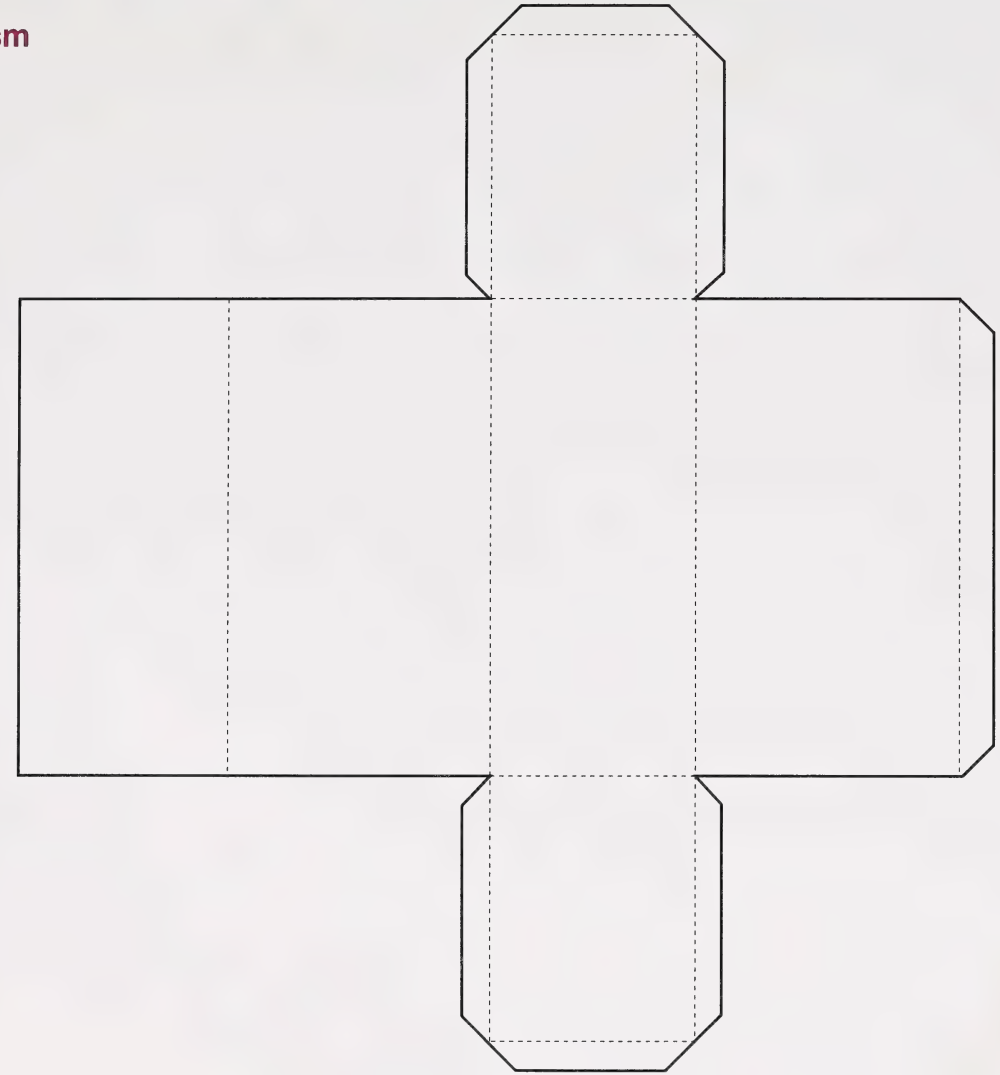
Pythagorean Puzzle 1



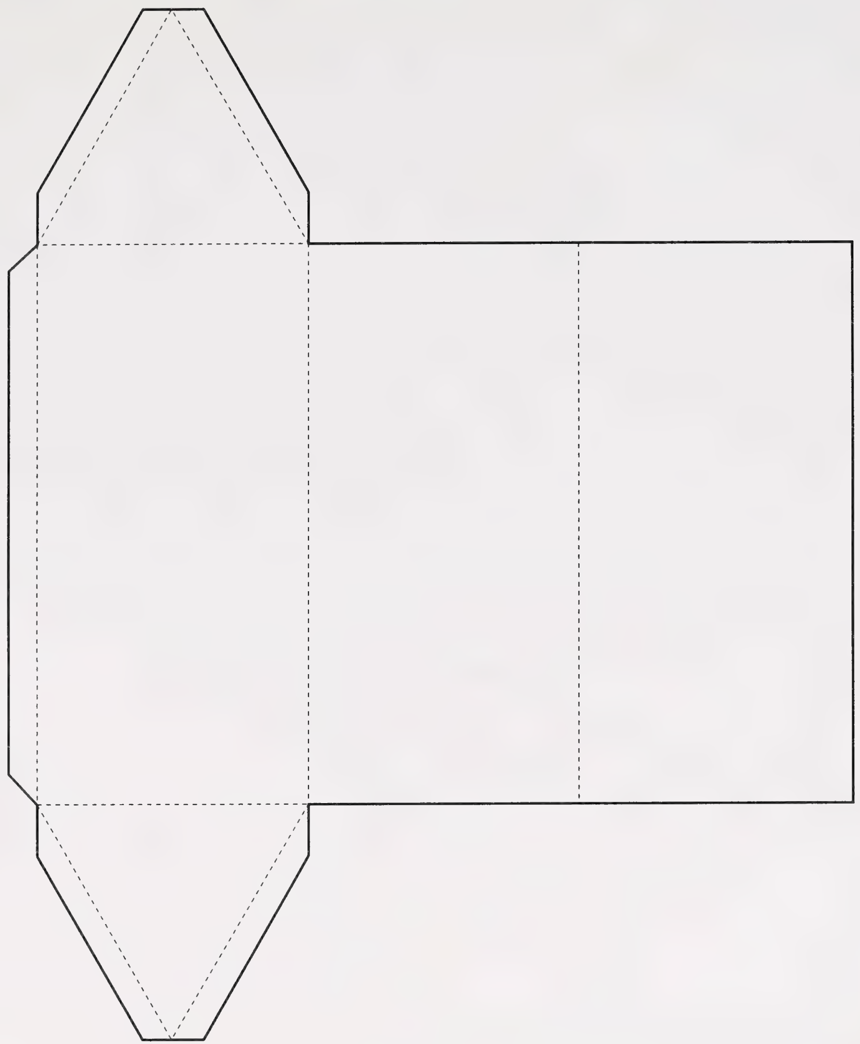
Pythagorean Puzzle 2



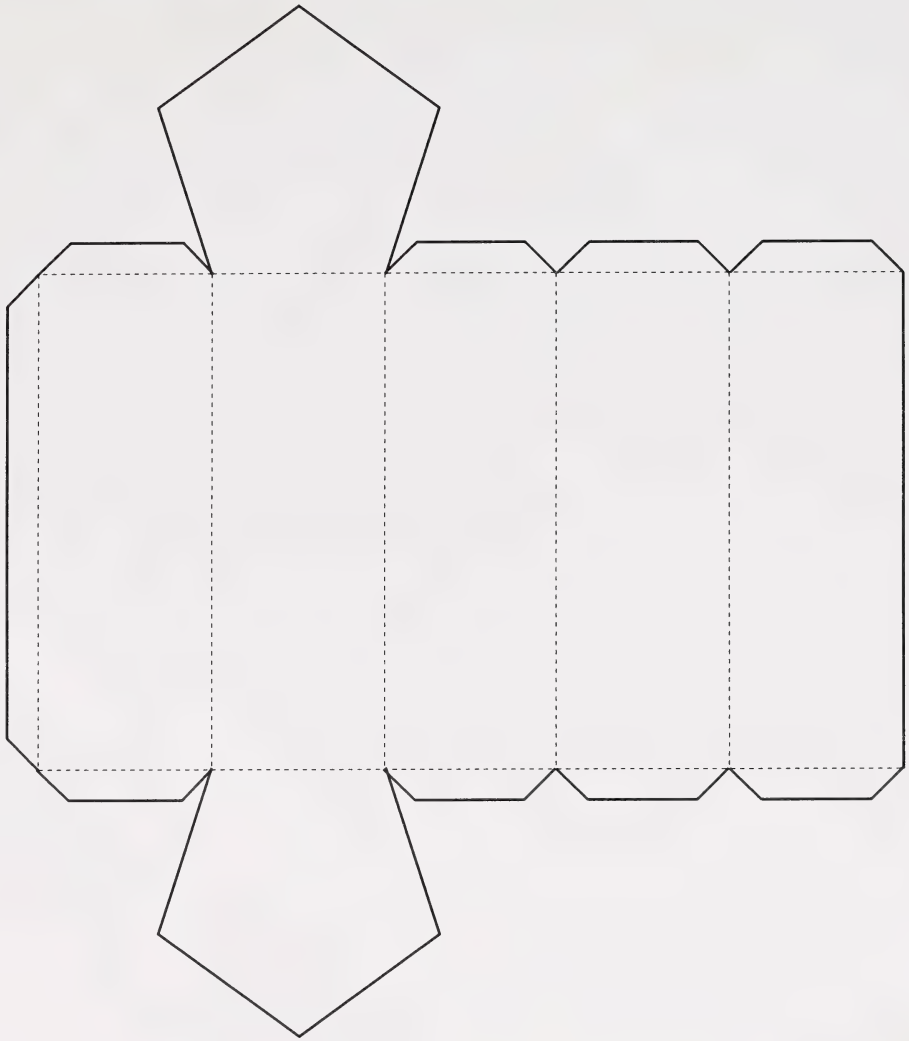
Rectangular Prism



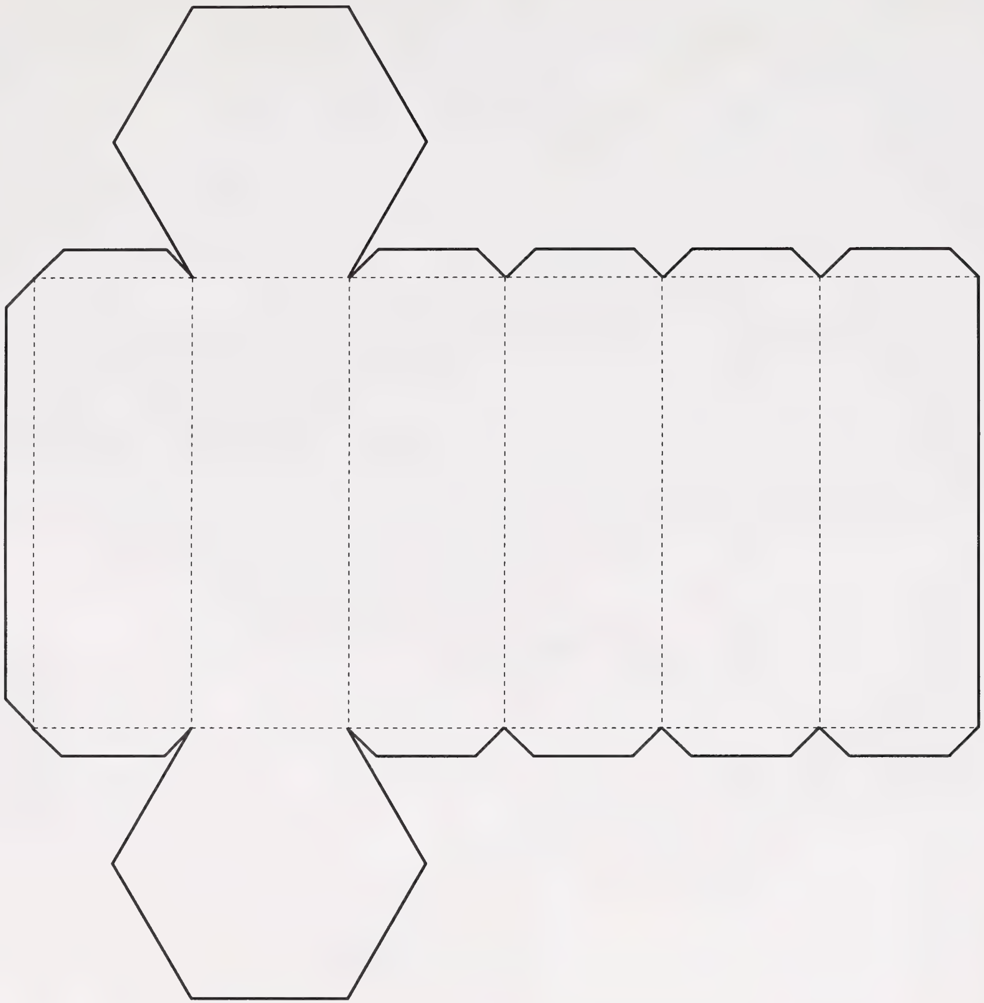
Triangular Prism



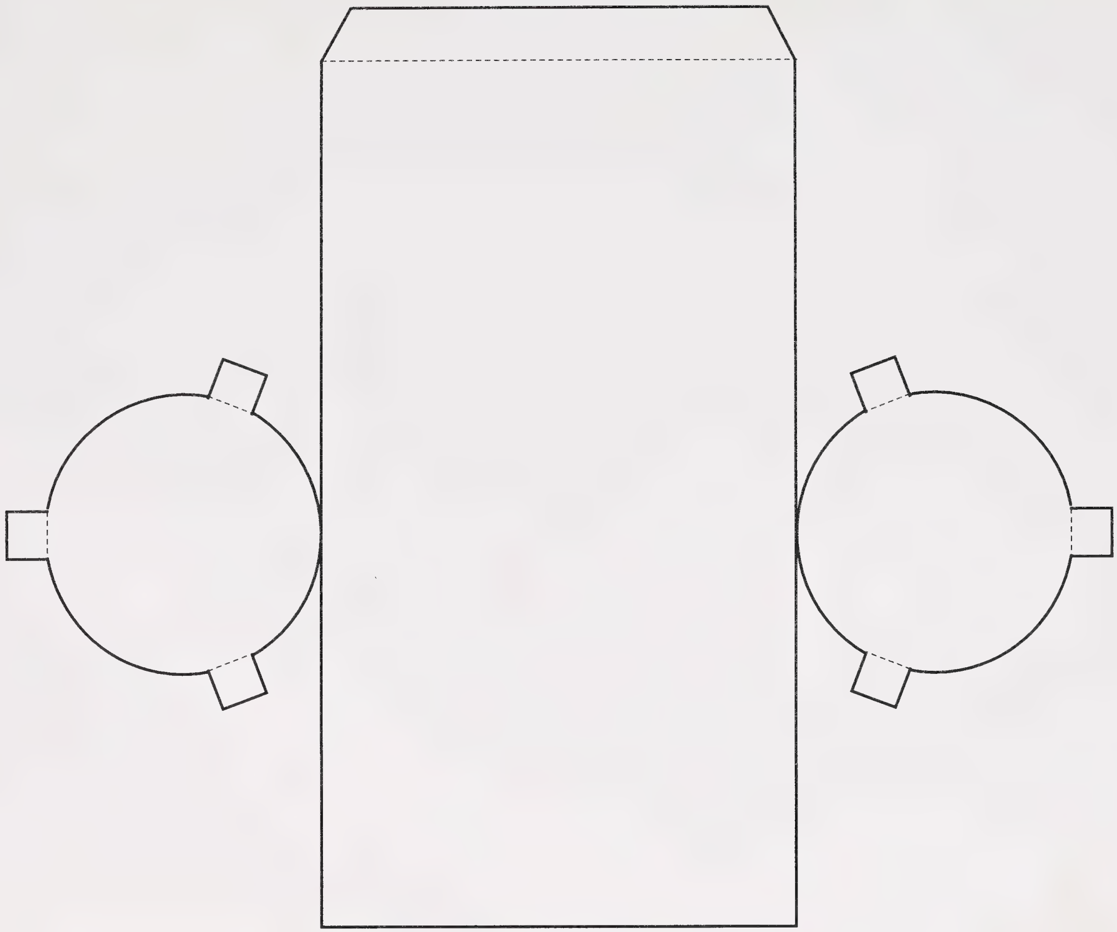
Pentagonal Prism



Hexagonal Prism



Cylinder



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Mathematics 9

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Module 6

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